



# On the universality of $I$ –Love– $Q$ relations in magnetized neutron stars

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## ABSTRACT

Recently, general relations among the quadrupole moment ( $Q$ ), the moment of inertia ( $I$ ) and the tidal deformability (Love number) of a neutron star were shown to exist. They are nearly independent of the nuclear matter equation of state and would be of great aid in extracting parameters from observed gravitational waves and in testing general relativity. These relations, however, do not account for strong magnetic fields. We consider this problem by studying the effect of a strong magnetic field on slowly rotating relativistic neutron stars and show that, for simple magnetic field configurations that are purely poloidal or purely toroidal, the relation between  $Q$  and  $I$  is again nearly universal. However, different magnetic field geometries lead to different  $I$ – $Q$  relations, and, in the case of a more realistic twisted-torus magnetic field configuration, the relation depends significantly on the equation of state, losing its universality.  $I$ –Love– $Q$  relations must thus be used with very great care, since universality is lost for stars with long spin periods, i.e.  $P \gtrsim 10$  s, and strong magnetic fields, i.e.  $B \gtrsim 10^{12}$  G.

**Key words:** gravitational waves – magnetic fields – MHD – binaries: general – stars: neutron.

## 1 INTRODUCTION

Neutron stars (NSs) offer a unique opportunity to investigate the state of matter at high densities, allowing us to probe aspects of the strong interaction in conditions that cannot be reproduced with terrestrial experiments. The equation of state (EOS) of nuclear matter at such high densities is highly uncertain, and constraints can only be inferred indirectly by studying its imprint on the exterior properties of the star. For example, there have been recent efforts to use observations of X-ray bursters to simultaneously constrain the mass and the radius of the NS (Steiner, Lattimer & Brown 2010), while radio observations of pulsars, such as the double binary J0737-3039 (Burgay et al. 2003), could be able to put constraints on the moment of inertia. It is also likely that gravitational wave (GW) observations of binary NS inspirals with Advanced LIGO/Virgo (Harry et al. 2010), KAGRA (Somiya 2012) or the planned Einstein Telescope (Punturo et al. 2010) will allow for further constraints on the spin, quadrupolar deformation and tidal Love number of the star.

Recent work has shown that, in slowly rotating and weakly magnetized NSs, unique relations exist between the quadrupole moment, the moment of inertia and the tidal Love number. These relations are “universal”, as they are essentially independent of the EOS as first shown by Yagi & Yunes (2013b) and then confirmed by Maselli et al. (2013) and could be used to break degeneracies between parameters in GW signals. This would allow us, for example, to determine

NS spins, conduct tests of general relativity (Yagi & Yunes 2013a) and distinguish between NSs and strange stars (Urbanec, Miller & Stuchlík 2013; Yagi & Yunes 2013b).

In this Letter, we consider the effect of the stellar magnetic field on such universal relations. NSs are strongly magnetized stars, with magnetic fields at the surface inferred to be of up to  $10^{12}$  G for radio pulsars, and of up to  $10^{15}$  G for magnetars. It is well known (see, e.g. Chandrasekhar & Fermi 1953; Bocquet et al. 1995; Haskell et al. 2008; Ciolfi, Ferrari & Gualtieri 2010; Friebe & Rezzolla 2012) that a magnetized NS cannot be spherical, with deformations that can lead either to oblate or prolate shapes, and may be even larger than those due to rotation.

First of all, we will show that, although the influence of the EOS is weak for a given simple magnetic field configuration, different geometries of the field lead to a different relation between the quadrupolar deformation and the moment of inertia. Therefore, as soon as deformations are dominated by magnetic fields, the universality no longer holds. Most NSs in binaries are spinning fast enough that this is not the case. However, in slow enough systems, i.e. when the NS spin period is of the order of a few seconds, it is possible that the quadrupolar deformation may be dominated by magnetic effects. It is generally thought that the interior magnetic field of an NS may be much stronger than the surface field (Braithwaite 2009; Corsi & Owen 2011; Özel 2013). Recent calculations of equilibrium models with magnetic fields in a twisted-torus configuration support this view, showing that the internal field can be up to two orders of magnitude stronger than the external one, leading to very large deformations (Ciolfi & Rezzolla 2013). In slowly rotating stars, these magnetic deformations can easily dominate

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the quadrupole. Moreover, for twisted-torus configurations, the  $I$ – $Q$  relation depends on the EOS, further invalidating the universality.

As a result, the ‘universal’ relations found by Yagi & Yunes (2013b) and extended by Maselli et al. (2013) are not applicable to highly magnetized, slowly spinning NSs, for which they would lead to an erroneous determination of the GW parameters. On the other hand, measured deviations from the universal relations of Yagi & Yunes (2013b) and Maselli et al. (2013) may potentially be used to constrain the geometry of the NS internal magnetic field, which cannot be probed with standard electromagnetic observations.

## 2 FORMALISM

We calculate the relation between the quadrupolar deformation of the star,  $Q$ , and its moment of inertia,  $I$ . We present our results in terms of the dimensionless quantities  $\bar{I} \equiv I/M^3$  and  $\bar{Q} \equiv Q/(M^3\chi^2)$ , where  $M$  is the mass of the star, and  $\chi \equiv J/M^2$ ,  $J$  being the spin angular momentum of the star (Yagi & Yunes 2013b).<sup>1</sup> Note that the quadrupolar deformation is the result of a rotational part,  $Q_r$ , and a magnetic part,  $Q_m$ , i.e.  $Q = Q_m + Q_r$ , but the normalization of  $Q$  assumes that the star is always rotating, i.e. that  $\chi \neq 0$ . Already for non-rotating models, however,  $Q_m \neq 0$  in the presence of a magnetic field, and the natural quantity to use to obtain a normalization would thus be the magnetic energy. For simplicity, and to easily compare with previous results, we will continue to define  $\bar{Q}$  as  $Q/(M^3\chi^2)$ . Note that, for all practical purposes, the slowly rotating models considered here have essentially the same physical properties as the corresponding non-rotating ones.

In what follows, we briefly discuss the general-relativistic mathematical setups used for the calculation of  $\bar{Q}$  and  $\bar{I}$ , either within a perturbative approximation, or in a fully non-linear approach. To understand how a magnetic field can break the universality of the  $\bar{I}$ – $\bar{Q}$  scaling relations, however, it is instructive to first consider the much simpler Newtonian case. It is sufficient to consider the Newtonian results for a rotating polytropic star with polytropic index  $n = 1$  and polytropic constant  $\kappa = 4.25 \times 10^4 \text{ cm}^5 \text{ g}^{-1} \text{ s}^2$  (Haskell et al. 2008). At lowest order for a purely poloidal magnetic field, the scaling relation between the normalized quadrupole and moment of inertia is given by

$$\bar{Q} \approx 4.9 \bar{I}^{1/2} + 10^{-3} \bar{I} \left( \frac{B_p}{10^{12} \text{ G}} \right)^2 \left( \frac{P}{1 \text{ s}} \right)^2, \quad (1)$$

where  $B_p$  is the field at the pole and  $P$  the rotation period. The first term in equation (1) is due to rotation (i.e.  $\propto Q_r$ ), while the second one is due to the magnetization (i.e.  $\propto Q_m$ ). This term was not analysed by Yagi & Yunes (2013b) and Maselli et al. (2013). Similarly, for a purely toroidal field, the scaling relation is

$$\bar{Q} \approx 4.9 \bar{I}^{1/2} - 3 \times 10^{-5} \bar{I} \left( \frac{\langle B \rangle}{10^{12} \text{ G}} \right)^2 \left( \frac{P}{1 \text{ s}} \right)^2, \quad (2)$$

where  $\langle B \rangle$  is now the field averaged over the volume of the star.

Given the expressions in (1) and (2), we can make a number of remarks that will be valid also when considering the results in a general-relativistic framework. First, in the case of purely toroidal magnetic fields, the magnetic quadrupolar deformation is negative, thus corresponding to a prolate shape. Secondly, with this definition of  $\bar{Q}$  the results depend on the product  $B \times P$  and will thus be,

in general, ‘non-universal’, as this product will vary from system to system. We will thus investigate the effect of the EOS on the  $\bar{I}$ – $\bar{Q}$  scaling relation at fixed period  $P$ , and then study the effect of varying  $P$ . Finally, it is clear from the coefficients in (1) and (2) that the magnetic corrections are generally smaller than those associated with the rotation, and that magnetic effects will only dominate for long rotation periods and strong magnetic fields. Hereafter, we will focus on the  $\bar{I}$ – $\bar{Q}$  relation, since the corrections on the Love number would be of higher order and no formulation of the Love number for magnetized and rotating objects has been derived yet. It is clear, however, that a loss of universality in the  $\bar{I}$ – $\bar{Q}$  relation implies a loss of universality also in terms of the Love number.

Let us now consider stellar equilibria in full general relativity, but with magnetic fields that are either purely poloidal or purely toroidal. Configurations of this type have been extensively studied in the past (Bocquet et al. 1995; Cardall, Prakash & Lattimer 2001; Kiuchi & Yoshida 2008; Friebe & Rezzolla 2012). Equilibrium models even with ultrastrong magnetic fields can be readily computed via the publicly available LORENE library,<sup>2</sup> and we refer to Bocquet et al. (1995) (MAGSTAR code) and Friebe & Rezzolla (2012) for details on the numerical implementation in the case of purely poloidal and purely toroidal configurations, respectively. Although fully non-linear and simpler to compute, these purely poloidal or purely toroidal configurations are known to be dynamically unstable on an Alfvén time-scale (Markey & Tayler 1973). Furthermore, the occurrence of this instability has been verified in a number of recent non-linear general-relativistic simulations (Ciolfi et al. 2011; Kiuchi, Yoshida & Shibata 2011; Lasky et al. 2011; Ciolfi & Rezzolla 2012; Lasky, Zink & Kokkotas 2012).

Let us thus consider a more realistic field topology, the so-called twisted-torus. In these configurations, the magnetic field has both poloidal and toroidal components, with the toroidal being possibly much stronger than the poloidal surface field. No general-relativistic solution has yet been found for this configuration in a fully non-linear setup. Nevertheless, twisted-torus configurations have been explored extensively in recent years, either in Newtonian non-linear equilibria (Tomimura & Eriguchi 2005; Yoshida & Eriguchi 2006; Lander & Jones 2009), or within general-relativistic perturbative approaches (Ciolfi et al. 2009, 2010; Ciolfi & Rezzolla 2013). Following the latter approach, we consider the magnetic field as a perturbation on a background equilibrium solution of a non-rotating star with an EOS  $p = p(e)$ , where  $p$  is the pressure and  $e$  the energy density. Note that using non-rotating background models is a good approximation for rotation periods  $P \gtrsim 10 \text{ s}$  if the surface magnetic fields are  $\gtrsim 10^{12} \text{ G}$ . More precisely, we find that  $Q = Q_r$  for fully relativistic rotating stars with  $P \sim 10 \text{ s}$  and  $B = 0$  is comparable to  $Q = Q_m$  for a twisted-torus configuration with  $B_p \sim 10^{12} \text{ G}$ ; these can be taken as the critical periods and magnetic fields such that  $Q_r \sim Q_m$  for our twisted-torus configurations.

The azimuthal component of the vector potential  $A_\phi = \psi(r, \theta)$  must satisfy the Grad–Shafranov equation

$$\begin{aligned} & \frac{e^{-\lambda}}{4\pi} \left[ \partial_r^2 \psi + \frac{\partial_r \nu - \partial_r \lambda}{2} \partial_r \psi \right] + \frac{(\partial_\theta^2 \psi - \cot \theta \partial_\theta \psi)}{4\pi r^2} \\ &= -\frac{e^{-\nu}}{4\pi} \beta \frac{d\beta}{d\psi} - F(e + p) r^2 \sin^2 \theta, \end{aligned} \quad (3)$$

where the metric functions  $\nu(r)$  and  $\lambda(r)$  are determined from the background solution, while  $\beta(\psi)$  and  $F(\psi)$  are two arbitrary

<sup>1</sup> Different dimensionless normalizations are also possible, e.g. in terms of  $I/(MR^2)$  (Lattimer & Prakash 2001; Bejger & Haensel 2002; Urbanec et al. 2013).

<sup>2</sup> <http://www.lorene.obspm.fr>

functions that determine the field geometry ( $r$  and  $\theta$  are spherical coordinates). Once a solution for  $\psi$  is found by assuming regularity at the centre of the star and matching to the external vacuum solution (taken to be dipolar for simplicity, i.e.  $\psi(r, \theta) = -a_1(r)\sin^2\theta$ ), the magnetic field components are obtained by taking the curl of the vector potential  $A^i$ . To study twisted-torus configurations in which the toroidal magnetic field can be comparable to or stronger than the poloidal surface field, we consider the form suggested by Ciolfi & Rezzolla (2013) for the trial functions, i.e.

$$\beta(\psi) = \zeta_0 \psi (|\psi/\bar{\psi}| - 1) \Theta(|\psi/\bar{\psi}| - 1), \quad (4)$$

$$F(\psi) = c_0 [(1 - |\psi/\bar{\psi}|)^4 \Theta(1 - |\psi/\bar{\psi}|) - \bar{k}], \quad (5)$$

where  $\zeta_0$ ,  $c_0$  and  $\bar{k}$  are constants,  $\bar{\psi}$  is the value of  $\psi$  on the last closed-field line (tangent to the surface) and  $\Theta(x)$  is the Heaviside step function. Once the magnetic field configuration is determined, the new equilibrium configuration is found by perturbing the continuity equation  $\nabla_\mu(nu^\mu) = 0$  and the relativistic equations of hydrostatic equilibrium in the presence of electromagnetic fields

$$(e + p)u^\nu \nabla_\nu u_\mu + \partial_\mu p + u_\mu u^\nu \partial_\nu p = \frac{F_{\mu\nu} \nabla_\alpha F^{\nu\alpha}}{4\pi}, \quad (6)$$

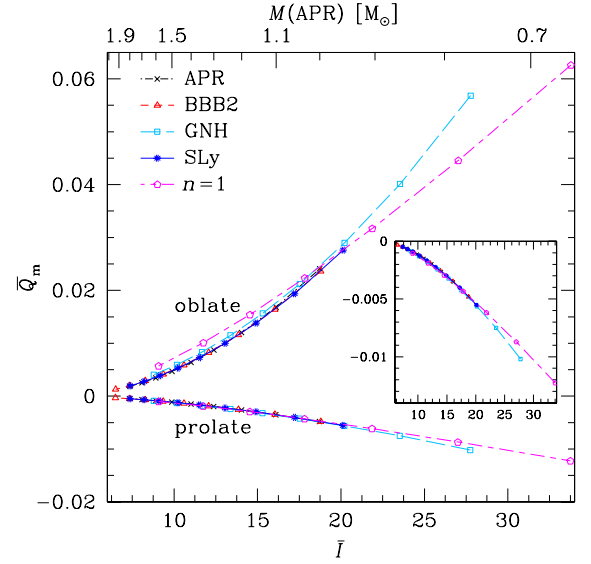
where  $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$  is the Maxwell tensor, such that  $F_{\mu\nu}u^\nu = 0$ , and  $\nabla$  denotes covariant derivatives with respect to the background metric.  $I$  and  $Q$  are then determined by matching the interior solution to the exterior metric of a slowly rotating star.

### 3 MAGNETIC DEFORMATIONS

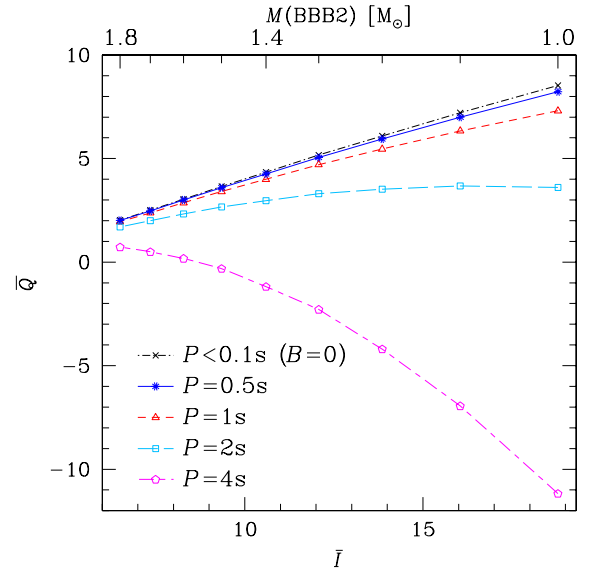
#### 3.1 Purely poloidal and purely toroidal configurations

We start our analysis by examining the case of an NS endowed with a purely poloidal magnetic field. As mentioned above, such a configuration is known to be unstable, but represents an adequate first step towards examining the more realistic twisted-torus configurations. We impose the field configuration using the prescription by Bocquet et al. (1995) and calculate the quadrupolar distortion of the star using the LORENE library. In Fig. 1, we show the magnetically induced  $\bar{Q}_m$  as a function of  $\bar{I}$ , for  $B_p = 10^{12}$  G. The data refer to non-rotating models, but a period of  $P = 2\pi$  s was used in normalizing  $Q$ . The results reported refer to a polytropic EOS with  $n = 1$  and  $\kappa = 14.6 \times 10^4 \text{ cm}^5 \text{ g}^{-1} \text{ s}^2$ , and to four realistic EOSs of cold nuclear matter, namely the APR EOS (Akmal, Pandharipande & Ravenhall 1998), the BBB2 EOS (Baldo, Bombaci & Burgio 1997), the GNH3 EOS (Glendenning 1985) and the SLy4 EOS (Douchin & Haensel 2001). In this case, all the deformations lead to  $\bar{Q}_m > 0$ , i.e. to an oblate shape, and it is easy to see that once the magnetic field configuration is fixed, the relation between  $\bar{I}$  and  $\bar{Q}$  is fairly ‘universal’ and depends only weakly on the EOS (the larger differences for the polytrope are mostly due to its inaccurate treatment of densities close to the crust).

A similar result holds in Fig. 1, when we consider purely toroidal magnetic fields calculated using the prescription of Friebe & Rezzolla (2012) and fixing the average magnetic field to  $\langle B \rangle = 10^{12}$  G. In this case, all deformations lead to a prolate shape, i.e.  $\bar{Q}_m < 0$ , but, as for purely poloidal fields, the relation appears to be EOS independent (the inset provides a magnified view for prolate models). It is important to stress, however, that such universality only holds once the magnetic field configuration and strength are fixed. In general, different magnetic field strengths lead to curves with different slopes, depending on the product  $\langle B \rangle \times P$ .



**Figure 1.** Dimensionless magnetically induced quadrupole deformation,  $\bar{Q}_m$ , as a function of the dimensionless moment of inertia,  $\bar{I}$ , for NSs with a purely poloidal magnetic field  $B_p = 10^{12}$  G (oblate configurations,  $\bar{Q}_m > 0$ ), and for NSs with a purely toroidal magnetic field  $\langle B \rangle = 10^{12}$  G (prolate configurations,  $\bar{Q}_m < 0$ ). The stellar mass for the APR EOS is shown in the upper x-axis as a reference; the models are non-rotating but  $P = 2\pi$  s was used for  $\bar{Q}$ .



**Figure 2.** Relation between  $\bar{Q}$  and  $\bar{I}$  for a purely toroidal field with an internal field strength of  $\langle B \rangle = 10^{14}$  G, for various rotation rates and the BBB2 EOS. The stellar mass is shown in the upper x-axis. Note that the slope of the curve changes, increasing with the rotation rate as the star goes from being prolate to being oblate.

The dependence on  $\langle B \rangle \times P$  is evident in Fig. 2, where the total quadrupole  $\bar{Q}$ , i.e. including rotational deformations, is shown as a function of  $\bar{I}$  for various rotation rates and an average magnetic field  $\langle B \rangle = 10^{14}$  G. Given the near universality of the results discussed previously, we restricted the analysis to the BBB2 EOS, but similar results hold for other EOSs. Note that as the rotation period  $P$  decreases from 4 s to  $< 0.1$  s, from bottom to top, the slope of the scaling relation increases as the star goes from being prolate to

being oblate. Different stars rotate at different rates and have different magnetic field strengths. The product  $\langle B \rangle \times P$  will thus change from system to system, leading to different  $\bar{I}$ – $\bar{Q}$  relations. For very short periods, however, the curves no longer show any influence of the magnetic deformation and agree with the  $\bar{I}$ – $\bar{Q}$  relation for an unmagnetized star, as the quadrupolar distortion is now dominated by rotational effects. As a result, for periods below  $\approx 0.1$  s the magnetic contribution to  $\bar{Q}$  becomes negligible.

### 3.2 Twisted-torus configuration

We can now advance our analysis by considering more realistic twisted-torus configurations. We use equations (4) and (5) to obtain twisted-torus configurations with internal toroidal-to-total magnetic field energy ratio  $F \equiv E_{\text{tor}}/E_{\text{m}}^{\text{int}} = 50$  per cent, a surface (polar) magnetic field strength of  $5 \times 10^{12}$  G, and  $\bar{k} = 0.25$  (cf. equation 5); while the surface field is fixed, the interior one changes from configuration to configuration, but is always  $\langle B \rangle \gtrsim 10^{13}$  G.

The results for the  $\bar{I}$ – $\bar{Q}$  relation are summarized in the left-hand panel of Fig. 3, using different EOSs and a  $P = 10$  s normalization, (cf. Fig. 1). In this case, the change in EOS has a considerable impact on the current distributions, and this leads to significant differences in both  $\bar{Q}$  and  $\bar{I}$ . Although the realistic EOSs show a behaviour that does not produce large variations (especially when compared to the  $n = 1$  polytrope, shown fully in the inset), the near universality of the  $\bar{I}$ – $\bar{Q}$  relation found in Fig. 1 for purely toroidal or poloidal configurations is not recovered. Note that although small,  $\bar{Q}_{\text{m}}$  is effectively comparable with  $\bar{Q}_{\text{r}}$  at these rotation rates and magnetic field strengths.

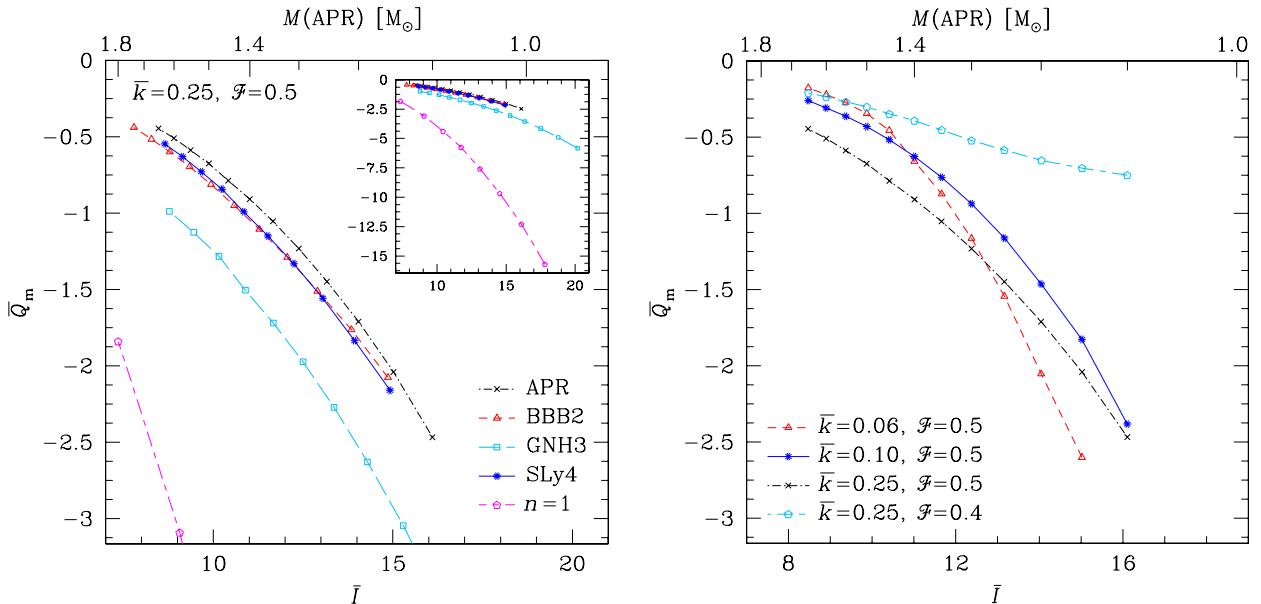
In addition, the results depend sensitively on the changes of the overall poloidal-to-toroidal field ratio and on the prescription for the currents inside the star. These changes lead to significantly different geometries and relations between  $\bar{Q}$  and  $\bar{I}$ , as shown in the right-hand panel of Fig. 3. In this panel, the different sequences refer to different values of the parameter  $\bar{k}$ , which in turn lead to

different internal field strengths. Note that once rescaled to the same polar magnetic field strength and rotation period, the twisted-torus deformations (cf. Fig. 3) are larger than those obtained with a purely poloidal field (cf. Fig. 1) and the shape of the star is prolate, rather than oblate. This is due to the strong internal toroidal component of the magnetic field, which dominates the distortion for slowly rotating NSs. Finally, as comparison we also show in the right-hand panel of Fig. 3 a sequence still having a surface magnetic field strength of  $5 \times 10^{12}$  G and  $\bar{k} = 0.25$ , but where 40 per cent of the magnetic energy is in the (internal) toroidal magnetic field (i.e.  $F = 0.4$ ). Also in this case the new curve has a different slope, as the poloidal contribution counters the toroidal one, leading to less prolate configurations.

Note that we have not included the effect of superconductivity in our analysis. The protons in the outer core are, however, expected to form a type II superconductor, and this can substantially alter the dynamics of the system, as the magnetic field will be expelled from the bulk of the fluid and confined to flux tubes. Simple estimates suggest that the quadrupole for a superconducting star,  $Q_{\text{s}}$ , is simply related to that of a ‘normal’ star,  $Q_{\text{n}}$ , by  $Q_{\text{s}} \approx Q_{\text{n}} H_1 / B$ , where  $H_1$  is the lower critical field for superconductivity, expected to be around  $H_1 \approx 10^{14}$  G (Jones 1975; Easson & Pethick 1977). While this simple scaling is approximately true for purely poloidal and toroidal magnetic fields, leading to even larger deformations than those discussed so far, the situation for general mixed poloidal/toroidal fields is generally more involved (Lander 2013a,b) and will be the focus of future work.

### 4 CONCLUSIONS

We have shown that once a purely poloidal or a purely toroidal magnetic field configuration is fixed, the relation between the normalized magnetic quadrupole  $\bar{Q}$  and the normalized moment of inertia  $\bar{I}$  is nearly ‘universal’ and depends only weakly on the EOS, in agreement with similar conclusions reached by Yagi & Yunes



**Figure 3.** *Left-hand panel:* relation between  $\bar{Q}_{\text{m}}$  and  $\bar{I}$  for twisted-torus configurations with  $F = 0.5$ ,  $B_{\text{p}} = 5 \times 10^{12}$  G,  $\bar{k} = 0.25$  and  $P = 10$  s. Note that the star is prolate because of the strong internal toroidal field, and that the  $\bar{I}$ – $\bar{Q}$  relation is no longer recovered. *Right-hand panel:* the same as in the left-hand panel for the APR EOS, with different sequences referring to different values of  $\bar{k}$  or different fractions of the toroidal energy. In both panels, the stellar mass for the APR EOS is shown in the upper x-axis as a reference.



(2013b) and Maselli et al. (2013) in the absence of magnetic fields. However, if a more realistic twisted-torus configuration is considered, in which poloidal and toroidal components coexist, the field configuration itself depends on the EOS and could lead to significant differences also in the  $\bar{I}$ – $\bar{Q}$  relation for different EOSs. In general, different magnetic field geometries and/or strengths could lead to a different relation, even for the same EOS. Furthermore, already the Newtonian estimates (1) and (2) show that the value of  $\bar{Q}$  depends also on the ratio between magnetic and rotational energies, thus differing from star to star.

Naturally, a departure from universality in the  $\bar{I}$ – $\bar{Q}$ , and hence  $I$ –Love– $Q$ , relation will have strong implications for GW detection. It will no longer be possible to use the universal relations derived by Yagi & Yunes (2013a,b) and Maselli et al. (2013) to reduce the parameter space to search, and any parameter inferred from them will not be reliable unless it is known that the stars have weak magnetic fields, i.e.  $B \lesssim 10^{12}$  G, and are rotating at periods  $P \lesssim 10$  s. Above these periods and magnetic field strengths, the universality is lost when considering twisted-torus configurations.

Luckily, for most binary NS systems of interest for GW detection, it should be possible to use the  $I$ –Love– $Q$  relations, but not for all. For systems with more slowly rotating components this will require extreme care. As an example, let us consider the so-called ‘double’ pulsar PSR J0737–3039. This is a binary NS system in which both NSs are seen as radio pulsars (Lyne et al. 2004). Pulsar A has a spin period of  $P = 22.7$  ms and an estimated field strength of  $6.3 \times 10^9$  G. Pulsar B is much slower and has a spin period of  $P = 2.77$  s, with an estimated field strength of  $1.2 \times 10^{12}$  G. The time to merger is estimated to be around 85 Myr, at which point the spin period of pulsar B will have slowed down to  $P \approx 3.9$  s (assuming standard electromagnetic spin-down and no field decay). The results of Ciolfi & Rezzolla (2013) suggest that a realistic NS could plausibly harbour a strong internal magnetic field (up to two orders of magnitude stronger than the surface field), potentially leading to a situation very similar to the one illustrated in Fig. 2, where for  $P \approx 4$  s, the value of the quadrupole deviates significantly from that of an unmagnetized rotating star, and the NS could even be prolate. In this particular system, the average quadrupole is dominated by the rotational contribution of pulsar A, but great care must be used in systems containing such slowly rotating stars.

Such deviations from a universal relation would also hinder any test of general relativity, as they would introduce many more parameters in the analysis, and deviations from the expected trend could be prescribed to an unobserved strong interior magnetic field component. On the other hand, independent measurements of the different quantities (such as  $\bar{Q}$  and  $\bar{I}$ ) could lead to the identification of a strong internal magnetic field. We note that to leading order in the post-Newtonian analysis, the magnetic field would not affect the tidal deformability (i.e. the Love number), but it would impact on higher order corrections. The presence of a magnetic field, in fact, selects a preferred direction in space. As a result, the deformability of the star under an external tidal field is affected in a way which depends on the magnetic field strength, on its topology, and, ultimately, on the EOS. These ‘orientation corrections’, which could be misinterpreted as a highly multipolar magnetic field, will affect the emitted GWs in a way which has so far not been quantified. Neglecting these corrections could lead to an erroneous determination of the system parameters. It is thus essential that the  $I$ –Love– $Q$  relations are used with great care, ensuring that the spin of the stars

is sufficiently rapid and the magnetic field sufficiently weak, so that the relations can be applied with confidence.

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