# Chapter 19

# Subjects omitted in this course

As stated in the introduction, it would not be possible to include the whole of relativity in a book of manageable size. We chose to go into several selected topics, but omitted some other topics completely or nearly so. This short chapter is a list of the topics we omitted, with some suggestions to the readers for further reading. The suggested reading includes only textbooks and monographs.

The following topics were covered inadequately or not at all:

1. **Gravitational waves.** Speaking most generally, a gravitational wave is any gravitational field that propagates through space independently of matter. It may, but does not have to be periodic. There exists a large collection of exact solutions of Einstein's equations describing waves, for these see Ref. [33]. There exists also an elaborate theory of nearly-linear waves, a relatively good source for it is the book by Ohanian and Ruffini [210], and also the classic MTW course [52]. The theory of generation and detection of gravitational waves is worked out rather well, but progress in it is still going on, so the current knowledge can be gained only from papers. Finally, a sophisticated and elaborate experimental technology is already in place, but to keep up with this one has to attend conferences in addition to reading literature. The pioneer of the search for gravitational waves was J. Weber; his small book can be recommended to readers interested in the history of the subject [211].

2. **The Cauchy problem.** In each coordinate system, the set of Einstein's equations can be separated into those equations that contain at most the first-order time derivatives of the metric components, and those that are of second order in time. The former are limitations imposed on the initial data, the latter are the dynamical evolution equations. This approach makes it possible to discuss such problems as the global existence or nonexistence of solutions of Einstein's equations, horizons in general spacetimes and general principles of propagation of gravitational waves. The pioneering paper in this field was that by Arnowitt, Deser and Misner [212], that gave the name, ADM, to the whole

[210] H. C. Ohanian and R. Ruffini, *Gravitation and spacetime.* W. W. Norton & Company, New York – London 1994.

[211] J. Weber, *General relativity and gravitational waves.* Wiley Interscience, New York 1961.

[212] R. Arnowitt, S. Deser and C. W. Misner, in *Gravitation: An introduction to current research.* Edited by L. Witten. Wiley Interscience, New York 1962. Reprinted in *Gen. Relativ. Gravit.* **40**, 1997

approach. Refs. [52] and [166] are also good sources for this topic, and Ref. [106] makes elaborate use of it.

3. **Generating new stationary–axisymmetric solutions out of known solutions.** This is a very large field of activity. The Kerr solution resulted from the Einstein equations in consequence of a set of assumptions, not all of which had a clear interpretation. Changing these assumptions one can obtain other solutions. The first important step in this direction was made by Ernst [213], who reduced the Einstein equations for stationary axisymmetric vacuum spacetimes to one equation for one complex function, now called **the Ernst potential** (in a subsequent paper, this approach was extended to the Einstein–Maxwell equations [214]). Then it was discovered that these equations admit transformations of variables that lead from one solution to another, with a geometry that is not a coordinate transform of the initial one. On the basis of this finding, several **generating techniques** for obtaining new solutions were introduced. Meanwhile, the most famous among the next-generation stationary–axisymmetric metrics, the Tomimatsu–Sato solution [215], was found. An overview of the generating techniques can be found in Ref. [33], a more extended overview is the book by Belinskii and Verdaguer [216].

4. **The Penrose transform.** In this approach points of the spacetime that lie at infinity are mapped into finite points of another manifold, which in turn allows one to discuss values of functions instead of limits. This is a powerful tool, even though few spacetimes are known for which the Penrose transform was constructed explicitly. More on it can be found in the books by MTW [52] and by Hawking and Ellis [106].

5. **Cosmic censorship.** This subject was completely omitted in the present course. It is a lively paradigm, on which perhaps the best source is the book by Joshi [166].

6. **Experimental tests.** Apart from the very few classical and most basic tests, we did not really do justice to this subject. This is now a science in itself, with large groups of physicists involved in projects lasting many years. Apparently, there exists no up-to-date book on it. As a historical introduction to the subject, the old volume of proceedings of the Fermi school from 1972 can be recommended [217], a discussion of results and their meaning for the theory can be found in Ref. [4].

7. **Spinor methods.** This subject was also completely omitted in the present course. A large monograph [218] is available.

8. **Relativistic astrophysics.** We did not give a fair representation of the classical applications of relativity to astrophysics because we concentrated on the conceptual basis of relativity. The still most extended course on this subject are the two volumes by Zeldovich

---

(2008) with an editorial note by J. Pullin, *Gen. Relativ. Gravit.* **40**, 1989 (2008) and (auto)biographies of the authors: *Gen. Relativ. Gravit.* **40**, 1991, 1992 and 1993 (2008).

[213] F. J. Ernst, *Phys. Rev.* **167**, 1175 (1968).

[214] F. J. Ernst, *Phys. Rev.* **168**, 1415 (1968).

[215] A. Tomimatsu and H. Sato, *Phys. Rev. Lett.* **29**, 1344 (1972).

[216] V. Belinskii and E. Verdaguer, *Gravitational solitons.* Cambridge University Press, Cambridge 2001.

[217] *Experimental gravitation. Proceedings of the International School of Physics "Enrico Fermi", Course 56.* Edited by B. Bertotti. Academic Press, New York and London 1974.

[218] R. Penrose and W. Rindler, *Spinors and Space-Time.* Cambridge University Press 1984.

and Novikov [219, 220], more brief accounts can be found in Refs. [52] and [221].

9. **History of relativity.** This subject is treated rather superficially in most textbooks, and ours is no exception. That history is important and can be exciting is best attested by the classic book by A. Pais [222] – a detailed account of Einstein's life and scientific activities. Other important sources are the book by Mehra [5] that explains how relativity had been taking shape step by step, and the collection of original papers in which special and general relativity were created, Ref. [6]. Some important bits of history can be found in the monograph by R. H. Dicke [3].

10. **Special relativity.** We omitted this subject altogether because we assumed that special relativity is now part of all courses on electrodynamics and should be familiar to anyone setting out to study general relativity. Should any reader need to learn, we recommend the following sources: Synge's book [223] is an expert-level complete textbook. The book by Kopczyński and Trautman [224] is only in a small part devoted to special relativity, but it presents an enlightening geometrical approach that simplifies many problems. Equally enlightening is the textbook by Rindler [124]. Finally, special relativity in the context of electrodynamics is presented in a readable way by Jackson [225].

---

[219] Ya. B. Zeldovich and I. D. Novikov, *Relativistic astrophysics: Vol. I: Stars and relativity.* University of Chicago Press, Chicago 1971.

[220] Ya. B. Zeldovich and I. D. Novikov, *Relativistic astrophysics: Vol. II: The Universe and relativity.* University of Chicago Press, Chicago 1974.

[221] S. Weinberg, *Gravitation and Cosmology.* Wiley, New York (1972).

[222] A. Pais, *Subtle is the Lord... The science and the life of Albert Einstein.* Oxford University Press, Oxford 1982.

[223] J. L. Synge, *Relativity: The special Theory.* 2nd ed. North-Holland, Amsterdam 1965.

[224] W. Kopczyński and A. Trautman, *Spacetime and gravitation.* Państwowe Wydawnictwo Naukowe, Warszawa, and J. Wiley, Chichester – New York – Brisbane – Toronto – Singapore 1992.

[225] J. D. Jackson, *Classical electrodynamics.* Second edition. J. Wiley & Sons, Inc. 1975.