Magneto-hydrodynamic simulations of accretion flows

1. Presence of the magnetic field in the Universe

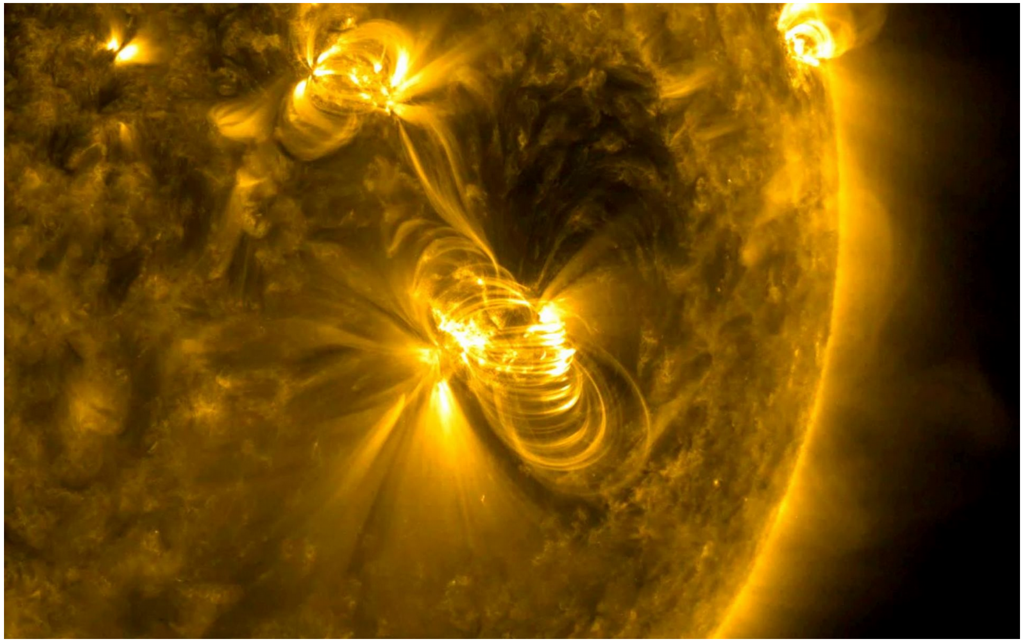
Magnetic field is actually everywhere.

(i) inside and around the compact objects:

MAGNETIC FIELD CLOSE TO THE SURFACE:

Sun	1 – 1000 G
Earth	1 G
Mars	10^{-3} G
Jupiter	0.1 G
White dwarfs - polars	10^{6} – 10^{7} G
Neutron stars in low mass binaries	10^{9} G
Pulsars	10^{12} G
Magnetars	10^{15} G

1 G = 10^{-4} Tesla



Sunspot – image from Solar Dynamic Observatory

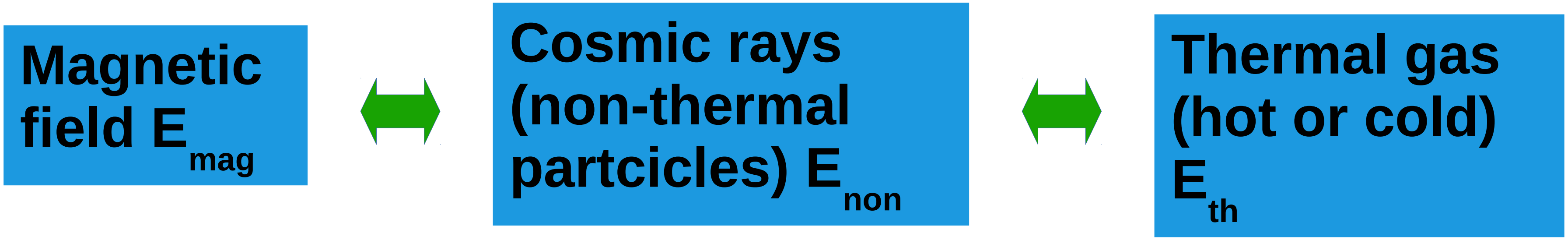
1. Presence of the magnetic field in the Universe

(ii) in the low density media

MAGNETIC FIELD IN THE LOW DENSITY MEDIA

Interstellar medium in Milky Way	$(0.5\text{-}2)10^{-5}$ G
Interstellar medium in M31	$4\text{x } 10^{-6}$ G
Intracluster medium in galaxy clusters	10^{-7} G
Intergalactic medium	10^{-9} G

Magnetic field in the low density medium is weaker in absolute terms but it is surprisingly strong if we compare the energy density in the magnetic field and energy density in other forms of matter (Spitzer 1978):



Magnetic field E_{mag}

Cosmic rays (non-thermal partcicles) E_{non}

Thermal gas (hot or cold) E_{th}

Equipartition of different forms of energy in the interstellar medium of Milky Way

The origin of the primordial magnetic field is still under discussion. If we some some magnetic field seed, magnetic field can be amplified by dynamo action.

2. The role of the magnetic field in accreting objects: motivation for detailed study

Magnetic field plays absolutely a key role in all aspects of accretion process:

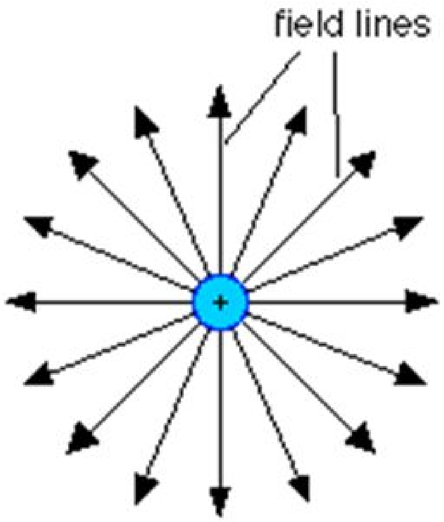
- It allows us to use hydrodynamical approach as it shortens the mean free path of particles (lecture 4)
- It provides a physical mechanism behind the α-viscosity (anomalous viscosity) in accretion flow
- It guides the inflow in sources like polars (white dwarfs in binary system with strong magnetic field) and X-ray pulsars (neutron stars with strong magnetic fields in in binary systems)
- It explains magnetar phenomenon
- It guides the outflow in the form of a collimated jet
- It is responsible for the formation of the hot compact corona emitting in X-rays in binaries and AGN
- It is responsible for the warm corona present in some soft states
- Uncollimated wind may also be (Sometimes? Always?) related to magnetic field
- Is it behind the outer standard disk/inner hot flow transition?
- …..

To do real progress in the description of the accretion progress we need a real progress in modelling the time evolution of the flow coupled with the magnetic field.

3. The magnetic field/electric field coupling

School summary:

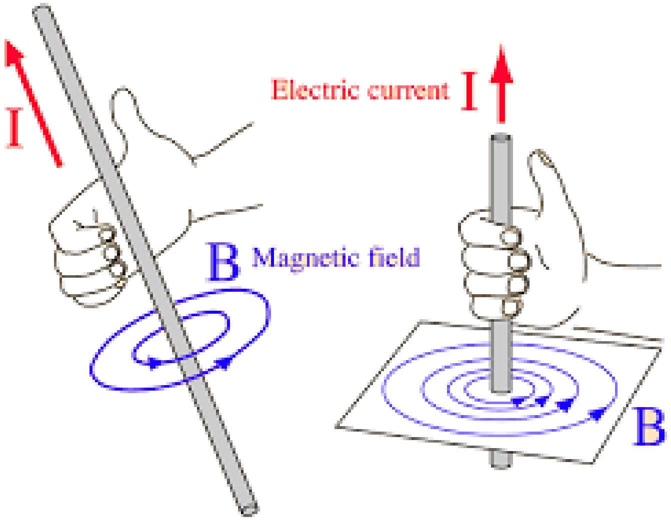
- Standing charge (electron or proton, for example) creates an electric field
- Moving charge (or current) creates also magnetic field
- So in the case of a single charge, having a magnetic or an electric field is a matter of reference frame
- Magnetic field does not perform work because acceleration is perpendicular to the direction of the field



The electric field from an isolated positive charge

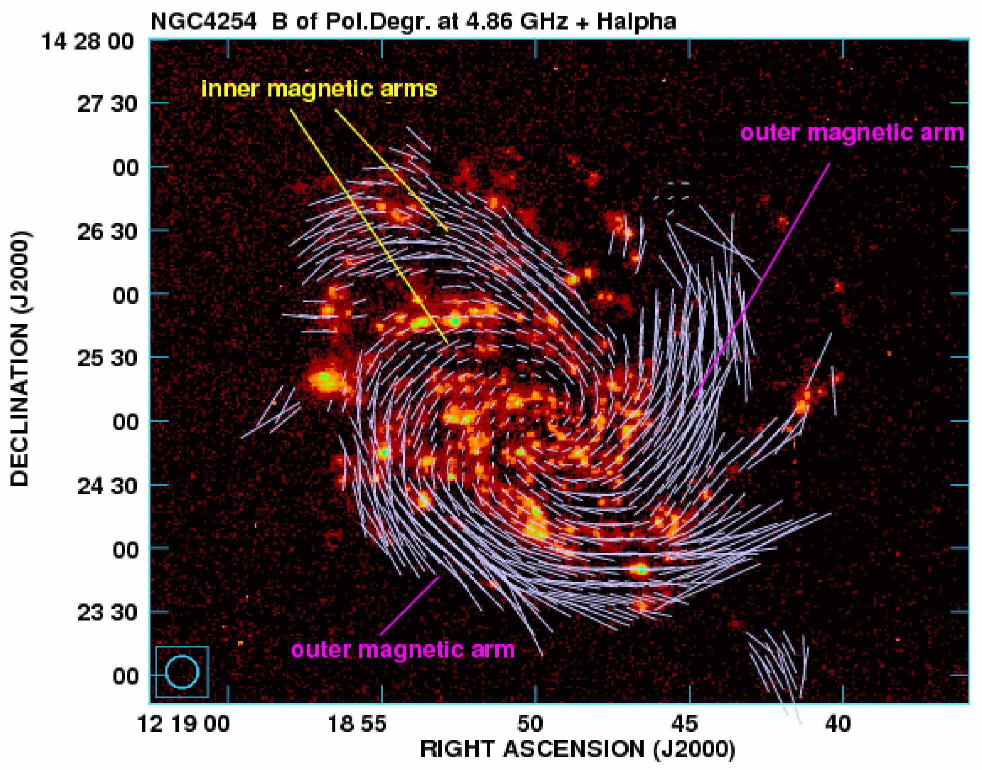
BUT

- Cosmic plasma is basically neutral so usually we do not have electric current but we do have magnetic fields
- In cosmic plasma usually either particles follow the field lines or field lines are frozen into the plasma and follows the magnetic field
- Magnetic field modifies the plasma motions and leads to energy dissipation so in a sense it ‘does perform work’



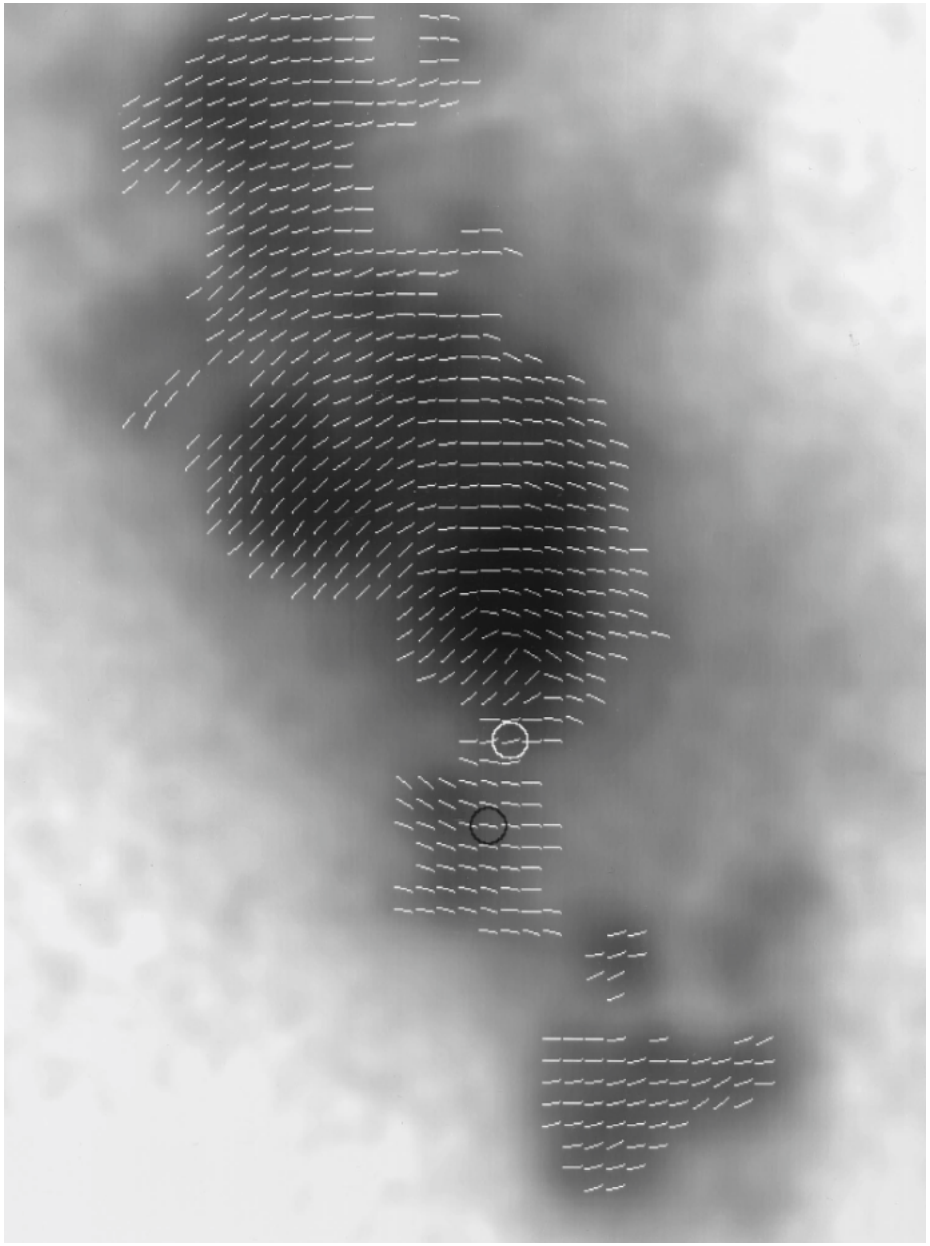
From http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magcur.html

4. Observational examples



High-resolution and high-sensitivity radio-polarimetric VLA observations of NGC 4254 from Effelsberg (Chyży et al. 2007)

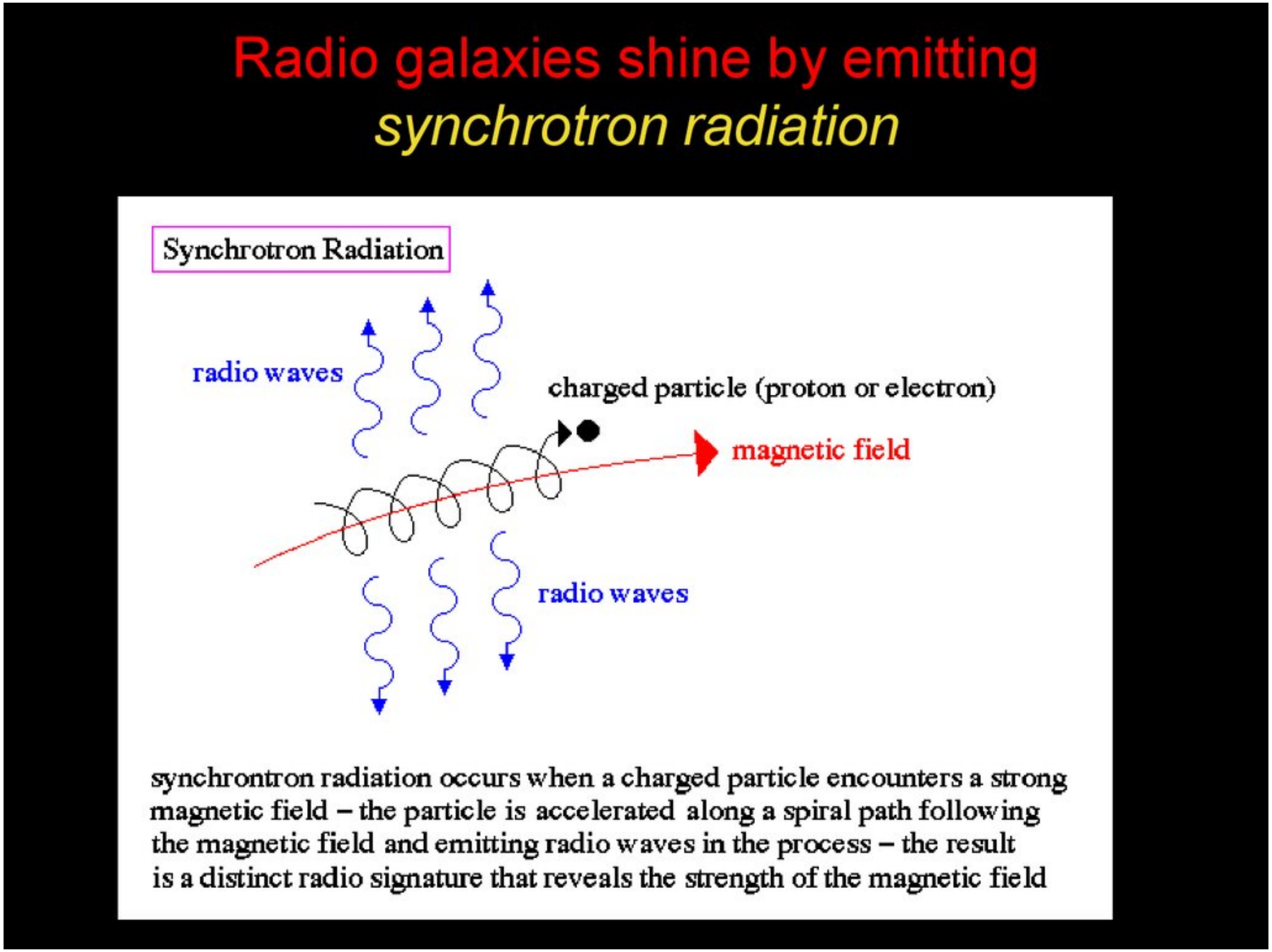
Properties of the magnetic fields are basically studied by observations of polarized synchrotron emission.



Polarization in the nucleus of NGC 1068 observed with HST (Capetti et al. 1995)

5. Motion of a particle in magnetic field

Some of the paradoxes mentioned before are easy to explain: we say that particles move ***along*** the field lines since we then neglect the details of the actual spiral motion which is essential as it provides the mechanism of synchrotron emission.



Radio galaxies shine by emitting
synchrotron radiation

synchrontron radiation occurs when a charged particle encounters a strong magnetic field – the particle is accelerated along a spiral path following the magnetic field and emitting radio waves in the process – the result is a distinct radio signature that reveals the strength of the magnetic field

From the lecture by Sandra Faber
https://ppt-online.org/326008

6. Generation of the magnetic field – Maxwell equations

Maxwell equations consist of four parts, with almost symmetric role of the electric and magnetic field. In SI units they read (after Chris Jones, http://www1.maths.leeds.ac.uk/~cajones/LesHouches/chapter.pdf)

Gauss law

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\epsilon}$$

Gauss law for magnetism

$$\nabla \cdot \mathbf{B} = 0$$

Faraday law of induction

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampère's circuital law (with Maxwell's addition)

$$\nabla \times \mathbf{B} = \mu \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Note:

- magnetic field does not have sources (no magnetic monopoles or currents)
- In order to change magnetic field we need an electric field

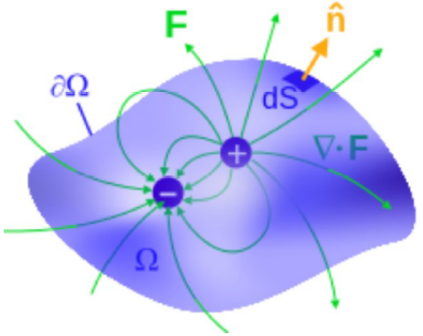
Here ρ_e is the charge density, **j** is the current (vector), ε is the dielectric constant, and μ is the permeability. Here we do not assume vacuum, but we assume that the medium is isotropic, i.e. parameters ε and μ are scalars instead of tensors (like for example in magnets).

Note: there are many system units which differ in various factors including 4π !

6. Generation of the magnetic field – Maxwell equations

Maxwell equations can be written in integral forms which can make their understanding easier. Here version from wikipedia, in vacuum

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega} \rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{B}\cdot \mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{l} = \mu_0\left(\iint_{\Sigma} \mathbf{J}\cdot \mathrm{d}\mathbf{S} + \varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{E}\cdot \mathrm{d}\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

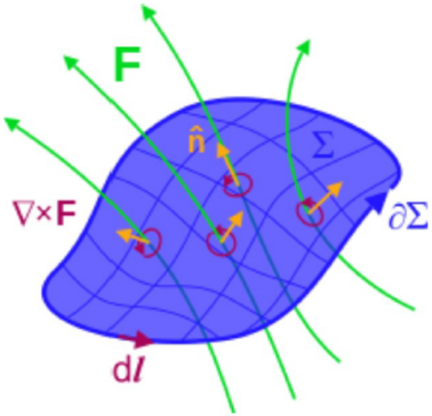


Volume Ω and its closed boundary $\partial\Omega$, containing (respectively enclosing) a source $(+)$ and sink $(-)$ of a vector field $\mathbf{F}$. Here, $\mathbf{F}$ could be the $\mathbf{E}$ field with source electric charges, but *not* the $\mathbf{B}$ field, which has no magnetic charges as shown. The outward unit normal is $\mathbf{n}$.

Summary:

Changing electric field induces magnetic field

Changing magnetic field induces electric field



Surface Σ with closed boundary $\partial\Sigma$. $\mathbf{F}$ could be the $\mathbf{E}$ or $\mathbf{B}$ fields. Again, $\mathbf{n}$ is the unit normal. (The curl of a vector field doesn't literally look like the "circulations", this is a heuristic depiction.)

7. Magnetohydrodynamics – basic assumption

(after Chris Jones, http://www1.maths.leeds.ac.uk/~cajones/LesHouches/chapter.pdf)

Gauss law

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\epsilon}$$

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$$\nabla \times \mathbf{B} = \mu \mathbf{j} + \xcancel{\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}}$$

Basic assumption of MHD:

- We drop the electric term in Ampère's equation

Here ρ_e is the charge density, **j** is the current (vector), ε is the dielectric constant, and μ is the permeability. Here we do not assume vacuum, but we assume that the medium is isotropic, i.e. parameters ε and μ are scalars instead of tensors (like for example in magnets).

These equations have to be complemented by another material-dependent equation relating the electric field and the current (Ohm's law):

j = σ **E**   for a medium at rest. Here σ is the conductivity.

7. Magnetohydrodynamics – basic assumption

From dimensional analysis of equations (L – length, t - time):

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \longrightarrow \frac{E}{L}=\frac{B}{t} \longrightarrow E=\frac{B\,L}{t}$$

Now we estimate the first and the third term in this equation:

$$\nabla\times\mathbf{B} = \mu\mathbf{j} + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\frac{B}{L} \qquad\qquad\qquad\qquad \frac{1}{c^2}\frac{B\,L}{t^2}$$

The ratio of the third term to the first term is equal

$$\frac{1}{c^2}\left(\frac{L}{t}\right)^2$$

So by neglecting the last term we assume that the system evolves much more slowly than the light travel time across the system.

Basic assumption of MHD:

- We drop the electric term in Ampère's equation

$$\nabla\times\mathbf{B} = \mu\mathbf{j} + \cancel{\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}}$$

7. Magnetohydrodynamics – basic assumption

We still need an equation combining the current and the electric field. For a medium at rest we have: $\mathbf{j} = \sigma \mathbf{E}$ but for a medium voving with velocity u, this equation should read

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}).$$

In analogy to the Lorentz force acting on a single electron: $\mathbf{F} = e(\mathbf{E} + \mathbf{u} \times \mathbf{B}).$

The simplest case of MHD is the dynamo equation

8. Dynamo equation

Since in astronomy in general we are not interested in the electric field, only in the magnetic field which is directly measured we eliminate the electric field explicitly present in equation for the magnetic field evolution using Maxwell equations.

We start from Ohm's law

$$\boldsymbol{j} = \sigma\left(\boldsymbol{E} + u \times \boldsymbol{B}\right) \Longrightarrow \nabla \times \frac{\boldsymbol{j}}{\sigma} = \nabla \times \boldsymbol{E} + \nabla \times \left(u \times \boldsymbol{B}\right)$$

Combining with $\boldsymbol{\nabla} \times \mathbf{B} = \mu \mathbf{j}$ and $\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$$\eta = \frac{1}{\mu\sigma}$$

We finally get an equation just for the evolution of magnetic field:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) - \boldsymbol{\nabla} \times \eta(\boldsymbol{\nabla} \times \mathbf{B})$$

Or

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

if magnetitic diffusivity η is constant.

8. Dynamo equation: important limits

Let us concentrate on constant magnetic diffusivity η:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Generation term   Diffusion term

If there is no motion of the plasma (**u** = 0), magnetic field can only diffuse:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}.$$

In this limit the initial magnetic field spreads and finally disappears after inifite time. The opposite limit is known as **perfect conductor** (η = 0)

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{B}).$$

In this limit we say that the magnetic field **is frozen into the plasma**, does not spread, and it is generated with the maximum rate if the motion is appropriate.

The ratio of the two terms is determined by the dimensionless magnetic Reynolds number, which using again just dimensional quantities instead of differential equations gives

$$R_m = \frac{u\,L}{\eta}$$

9. Key achievement of the dynamo theory in the context of accretion disks

Dynamo theory explained the viscosity action postulated by Shakura & Sunyaev (1973). The key paper is Balbus & Hawley (1991) on magnetorotational instability.

Their basic set of equations:

$$\frac{d\ln\rho}{dt} + \nabla\cdot\boldsymbol{v} = 0\,,$$

$$\frac{d\boldsymbol{v}}{dt} + \frac{1}{\rho}\nabla\left(P + \frac{B^2}{8\pi}\right) - \frac{1}{4\pi\rho}(\boldsymbol{B}\cdot\nabla)\boldsymbol{B} + \nabla\Phi = 0\,,$$

$$\frac{\partial\boldsymbol{B}}{\partial t} - \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) = 0\,.$$

They use Lagrangian setup.

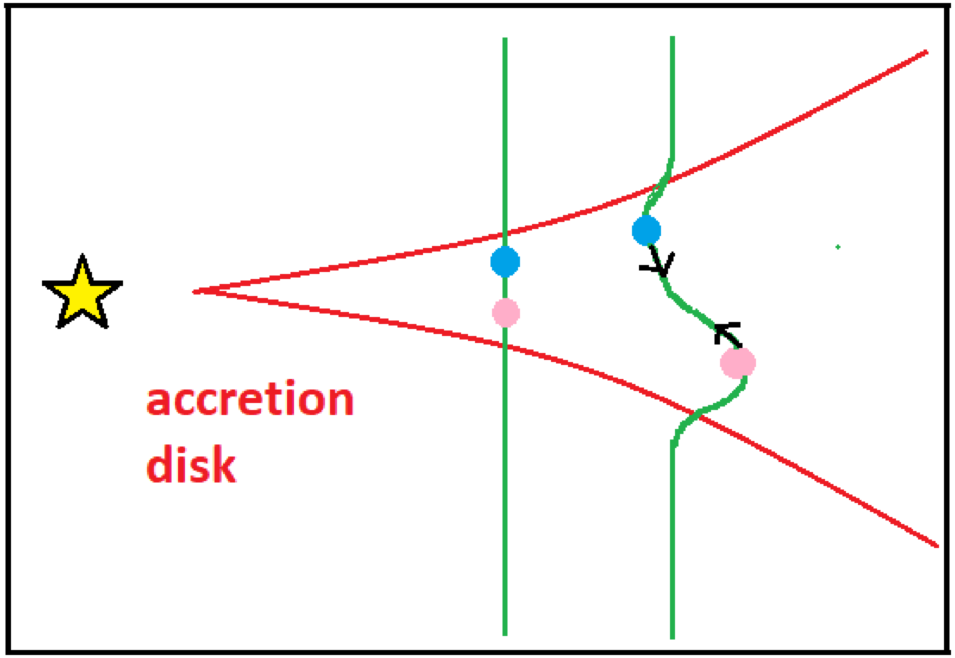
Thus they assume perfect conductivity, allow for external gravitational field. B enters also the equations of motion thougth the field-related pressure term and the Lorentz force term.

When Balbus & Hawley published their results, they did not know that the instability was known before (e.g. Velikhov, Chandrasekhar) but in different context.

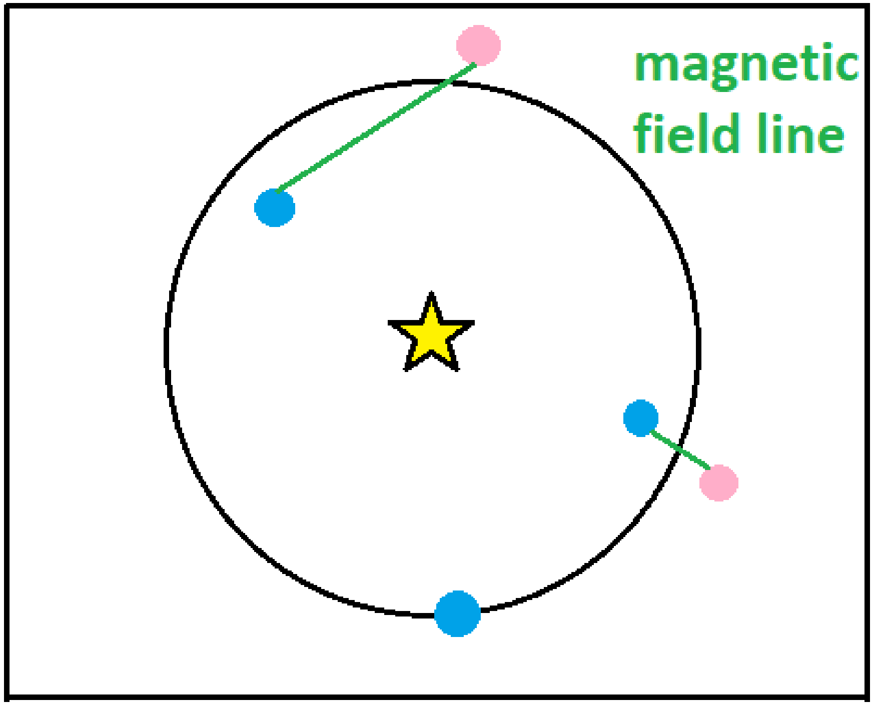
Note that:
- The paper is cited 3098 times
- The paper is actually analytical...

Linear perturbation theory showed that the solution for a Keplerian differentially rotating disk is highly unstable, and the rise time of the magnetic field strength is of the order of dynamical timescale.

9. Key achievement of the dynamo theory in the context of accretion disks



vertical cross-section of the disk with magnetic field and perturbed position of two blobs



Top view of the accretion disk showing how differential rotation streches the magnetic field lines.

The net result is that the blob at larger radius gains some angular momentum while the one at smaller radius looses the angular momentum.
The instability does not lead to ininite strength of the magnetic field but it leads to turbulence and saturation. This can be seen in numerical solutions. The net effect is the requested transport of angular momentum.

9. Key achievement of the dynamo theory in the context of accretion disks

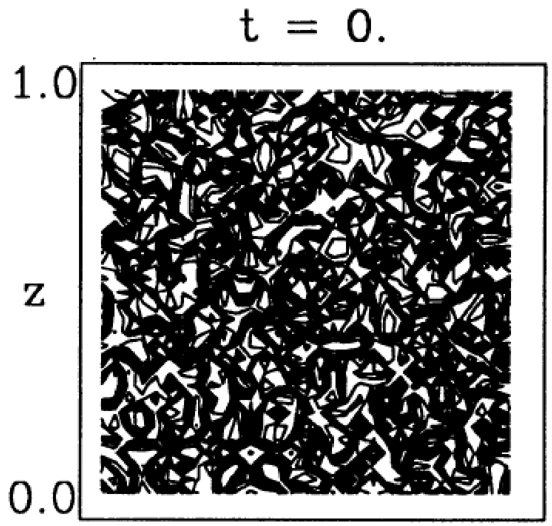
First 3-D numerical solutions: Hawley, Gammie and Balbus (1995) were done assuming politropic approximation and shearing-box approach, with no background gradient of pressure and density. They start from initial perturbations.

$$f(x, y, z) = f(x + L_x, y - q\Omega L_x t, z) \quad (x \text{ boundary}) ,$$

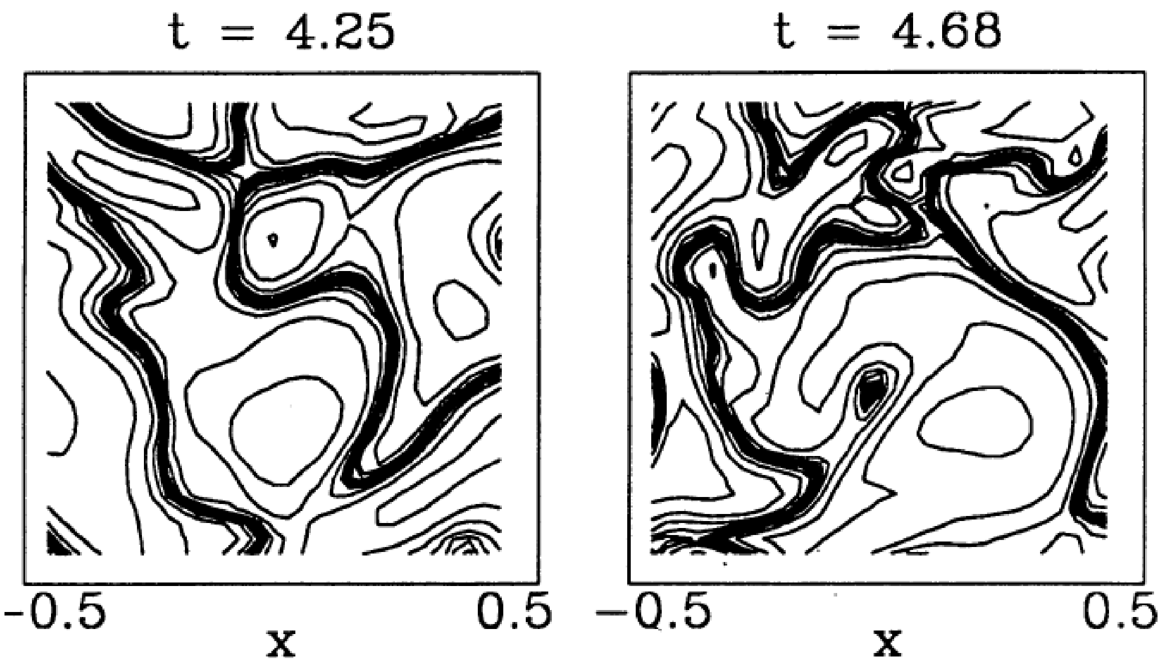
$$f(x, y, z) = f(x, y + L_y, z) \quad (y \text{ boundary}) ,$$

$$f(x, y, z) = f(x, y, z + L_z) \quad (z \text{ boundary}) .$$

$$v_y(x, y, z) = v_y(x + L_x, y - q\Omega L_x t, z) + q\Omega L_x \quad (x \text{ boundary}) .$$



The figures show the perturbed azimuthal velocity δv_y in z-x plane. Solution saturates.

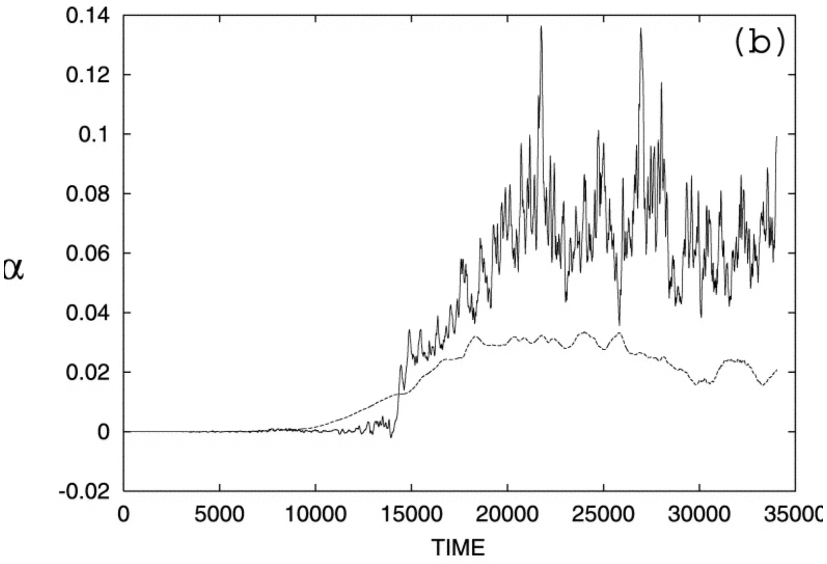


Shearing-box approximation does not describe the dynamics of the accretion flow due to periodic boundary conditions.

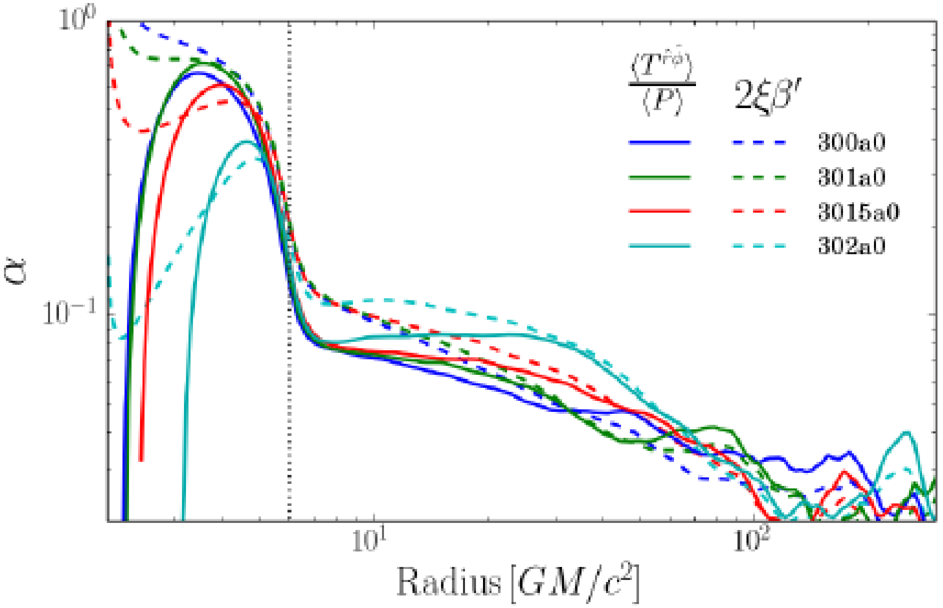
9. Key achievement of the dynamo theory in the context of accretion disks

What values of α do result from such simulations? This is related to the level at which saturations happens.

- Analytically: „We found that, for Keplerian disks, the MRI is stabilized for azimuthal Alfven speeds exceeding the geometric mean of the sound speed and the rotational speed” - from Pessah & Psaltis (2005); if I use this creterion plus the definition $\alpha = \frac{B^2}{8\pi P}$ plus the hydrostatic equation on Keplerian disk then I get something worrying like $\alpha = \frac{r}{8\pi H}$ the authors do not have that in the paper.
- Numerical example:



But this is from global 3-D simulations, Machida & Matsumoto (2003)



But this is from global 2-D simulations, Sadowski et al. (2015) which formally does not account for dynamo.

10. Global MHD simulations in the context of accretion disks

Global 2-D simulations do not properly represent the action of the dynamo.
Global 3-D simulation are enormously time-consuming, particularly in the version of GR (General Relativity) R(Radiative) MHD.

The results are very difficult to appreciate for non-experts since it required good understanding of the assumptions in each presented set of computations and they consist of

• Boundary conditions

The issue of the bounday conditions is easy to understand, they are analogous to all computations of for example stellar structure or accretion disk structure without magnetic field. Those are basically global parameters, like the black hole mass, external accretion rate, outter disk radius, inner boundary condition (different for a black hole and for a neutron star). Codes computing accretion disk instabilities, with stationary inflow to the outer disk give results depending on these assumptions, but independent on the initial mass distribution after the initial relaxation period.

• Initial conditions

When the external mass supply is not present, we also had an implicit dependence of the initial conditions when we did not discuss the magnetic field – in the case of Tidal Disruption Event, that was the initial mass and the initial radius, and they affected the overall time evolution.

In MHD we always have a strong dependence on the initial conditions.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

If B is initially zero, it will remain zero.

10. Global MHD simulations in the context of accretion disks

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

If B is initially zero, it will remain zero.

Without proper seed, the issue of geting strong global magnetic field would have some similarity to this book.

From wikipedia:

Baron Munchausen is a fictional German nobleman created by the German writer Rudolf Erich Raspe in his 1785 book ***„Baron Munchausen's Narrative of his Marvellous Travels and Campaigns in Russia”***. The character is loosely based on a real baron, Hieronymus Karl Friedrich, Freiherr von Münchhausen.

Baron Munchausen pulls himself out of a mire by his own hair.

10. Examples of MHD simulations in the context radiation pressure instability

As I mentioned in lecture 9, the disk dominated by the radiation pressure is unstable if we use αP_{tot} and stable if we assume αP_{gas}. In MHD we do not need to assume α so we should get a final answer.

Numerical MHD answers:

- Turner (2004) – radiation pressure dominated disk is stable
- Hirose, Blaes & Krolik 2009 – no thermal instability, possibly traces of viscous instability
- Jiang et al. (2014) – radiation pressure instability obtained in MHD simulations!
- Khosravi & Khesali (2017) – disks are mostly stable if the magnetic field is not too strong
- Jiang et al. (2019) – no radiation pressure instability again!

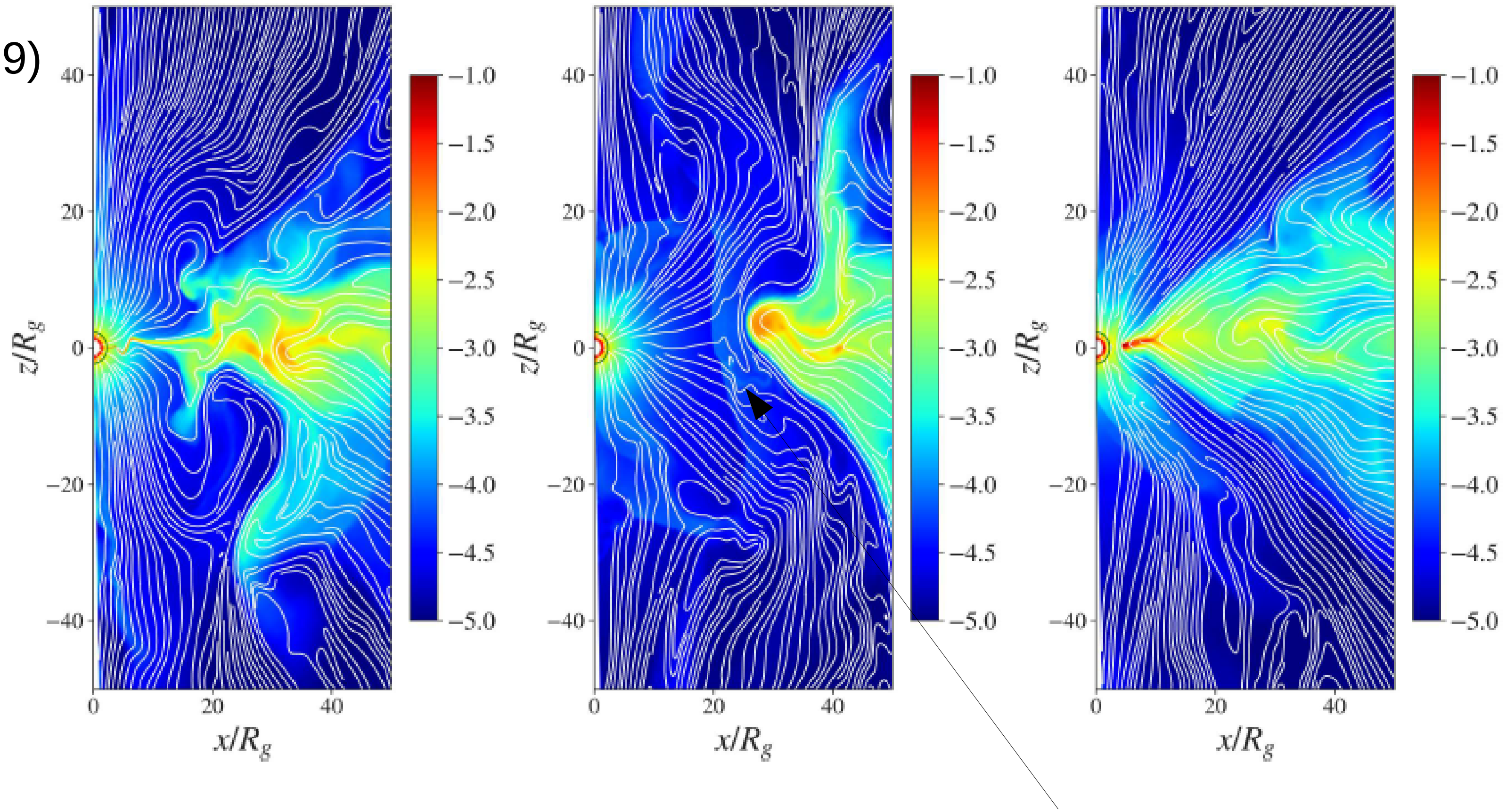
10. Examples of MHD simulations in the context of jet vs. wind outflow

Only about 10 % of AGN show strong well collimated jet outflows extending even far beyong the host galaxy. Jets are relativistic.

In galactic sources, during X-ray novae outburst, the source come through a jet period (hard state), then jet is quenched (soft state), and finally jet reappears (low hard state).

Do we see jets in 3-D simulations?

- Vourellis et al. (2019)



They perform GR MHD based on HARM-3D. They see periods of magnetically arrested disk. They see winds and jet. But

10. Examples of MHD simulations in the context of jet vs. wind outflow

- Vourellis et al. (2019)

They perform GR MHD based on HARM-3D. They see periods of magnetically arrested disk. They see winds and jet. But their jet is non-relativistic (Lorentz factor of less than 2, we need 10-20). And there is no trace of a compact dissipative corona either.

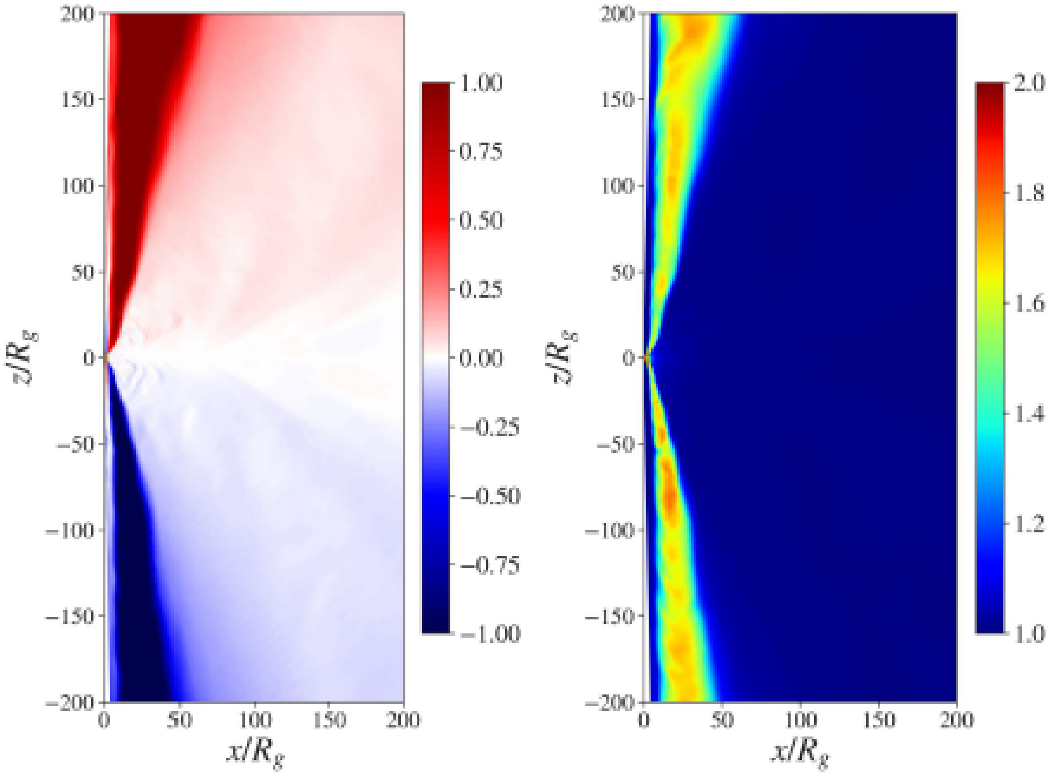


Figure 11. The vertical component of the velocity and the Lorentz factor on a sub-grid of 200 R_g.

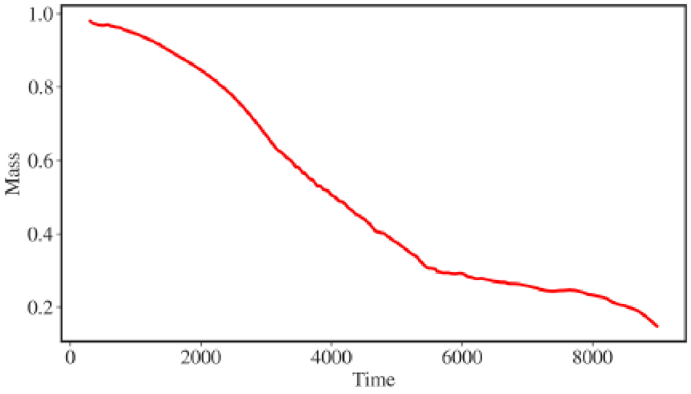


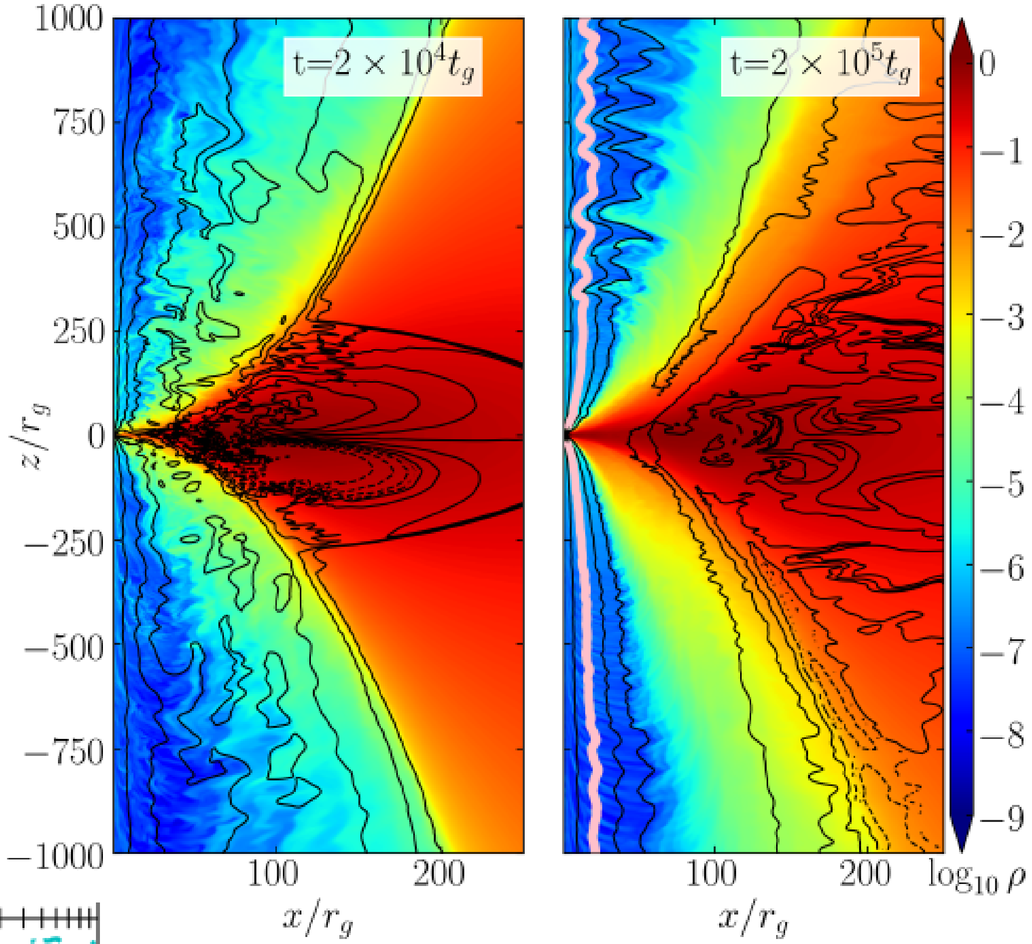
Figure 5. The evolution of the disk mass in our reference simulation measured in a reference area as described in the text. The mass is normalized to the initial disk mass.

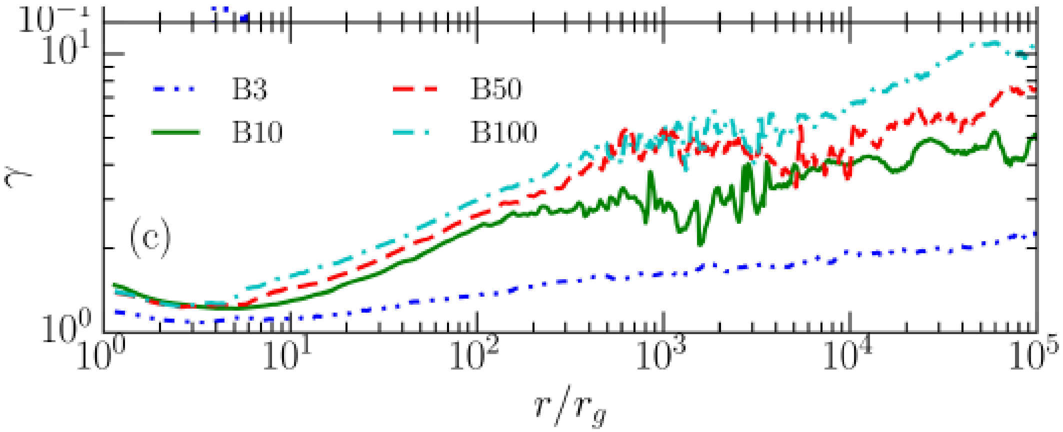
And as usually, the simulation ends up after emptying the disk since it is not clear how to supply the magnetic field. Supplying mass through the outer boundary seems (in principle) simple.

10. Examples of MHD simulations in the context of jet vs. wind outflow

- Chatterjee al. (2019)

They perform GR MHD based on HARM-2D and concentrate more on jet. They can get larger Lorentz factor (depending on the adopted magnetization) at large distances from the black hole.





They discuss the issue of the pinch instabilities, they affect the jet dynamics.

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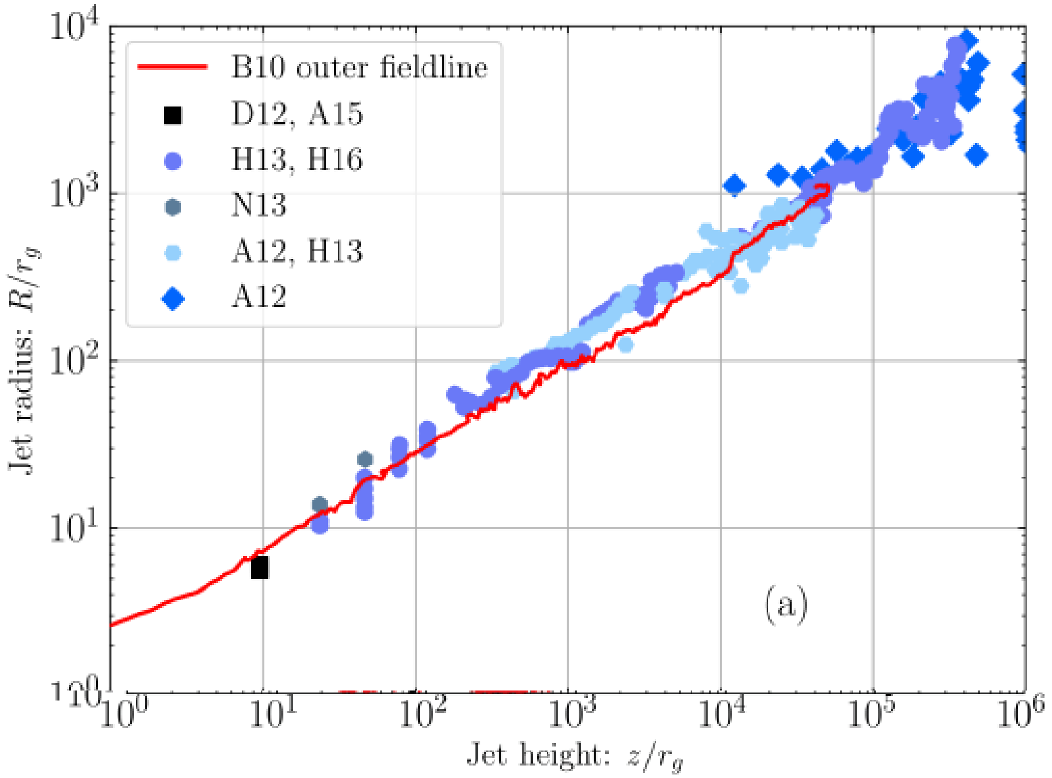
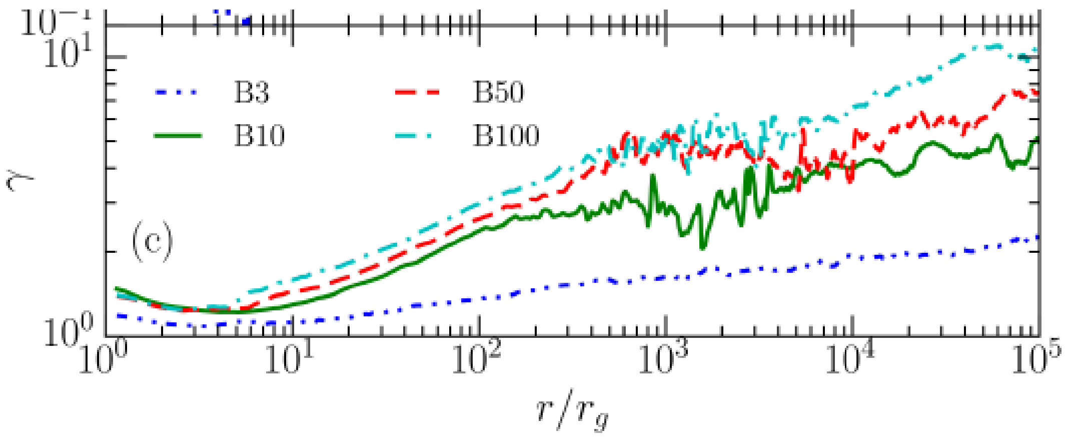




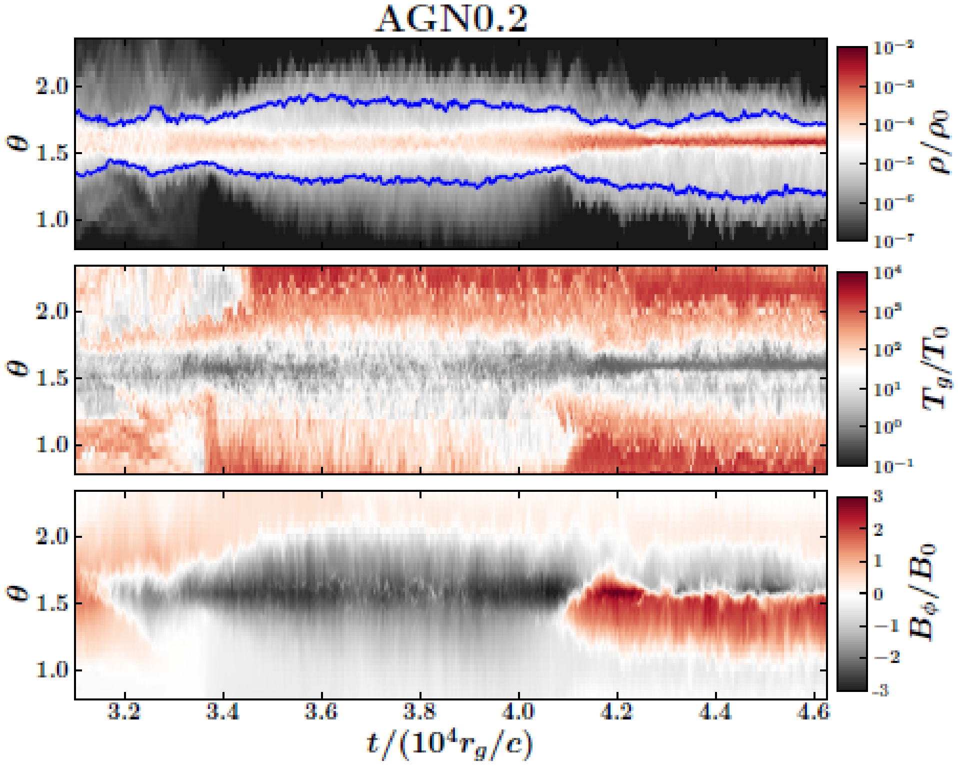
The width of the jet in the model compares favorably with the data points for M87.

10. Examples of MHD simulations

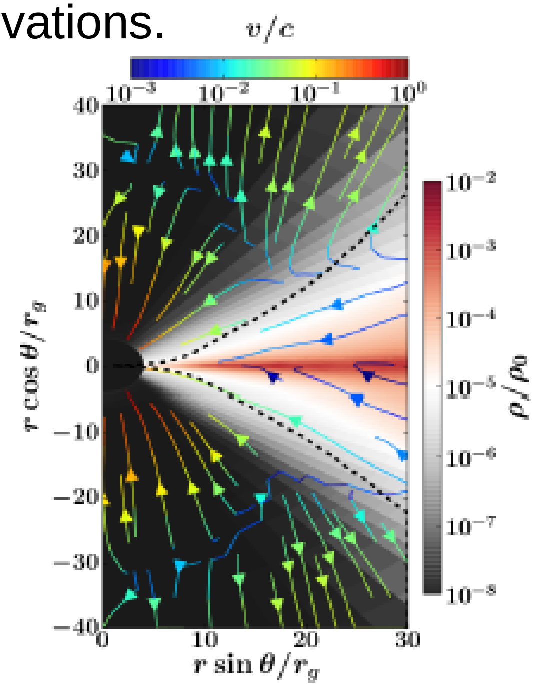
- Jiang al. (2019)

Previous simulation were mostly appropriate for a hot inner flow, as expected in M87, but this paper addresses the flow in almost standart but magnetized accretion disk. They consider two models giving finally 7% and 20% of the Eddington accretion rate, for a black hole mass 5 x 10^8 Ms.

New aspect of this simulation: they see a spontaneous division of the flow into cold disk flow and equally important coronal flow. This is probably required by observations.



Traces of radiation pressure instability ?



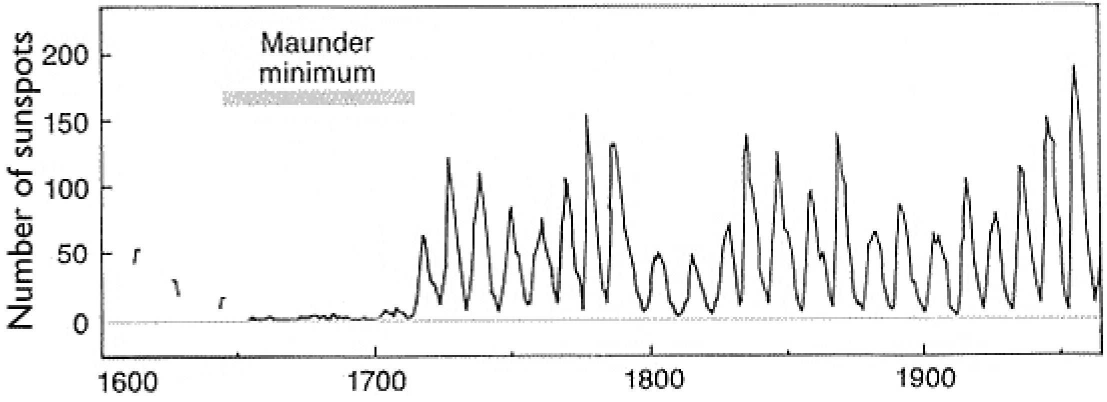
They do not see much outflow, and no jet, no compact corona. The coronal layer has the temperature up to 10^8 K, not enough for hard X-ray emission.

11. Large scale magetic field from local turbulent field

Jet formation requires large scale magnetic field, and large scale magnetic field in MHD simulations of AGN are present only when they are assumed, as coming from extrentitial medium (host galaxy). However, the most advances studies of the magnetic field are done so far in the context of the Sun.

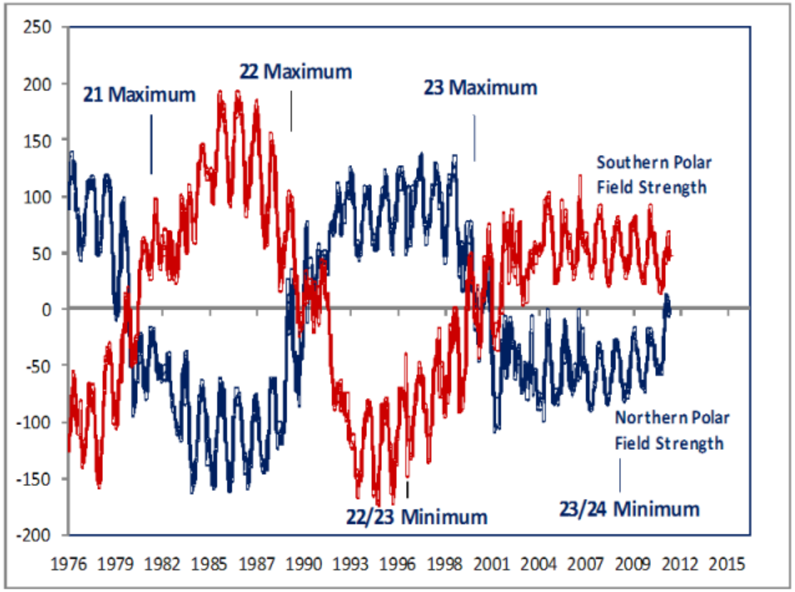
Observational facts:

- Sun rotates always in one direction
- Sun in general displays a large scale bipolar magnetic field which however changes the directon every 11 years (solar cycle)
- Magnetic field is related to rotation and convection



Historically, the solar cycle was studied at the basic of sunspot numbers. Recently Sun became more quiet, and right now we are in the solar minimum (no spots).

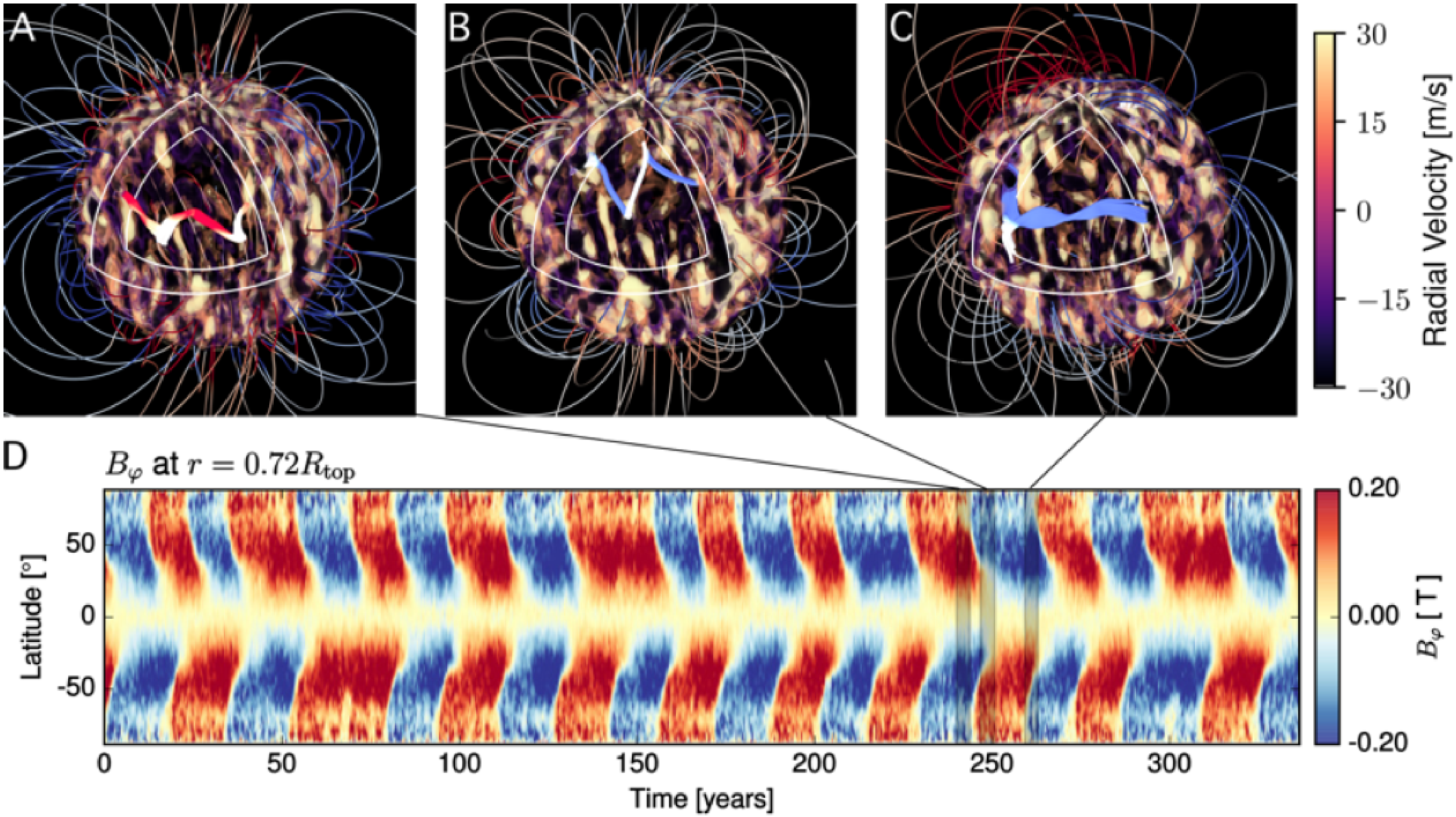
Every 11 years the direction of the magnetic field changes, despite the fact that the rotation of the Sun is always in the same direction.



https://www.thegwpf.com/waiting-for-the-next-sunspot-cycle-2019-2030/

11. Large scale magetic field from local turbulent field

The change of the magnetic field direction suggests that it indeed originates from the turbulent motion caused by convection. I do not understand how it happens but there are now simulations which can recover the solar cycle using the dynamo mechanism. The same mechanism can account for a random change of the direction of the magnetic field of Earth, well documented by geology (e.g. Petrelis & Fauve 2010).



Simulation of the solar cycle by Strugarek et al. (2017)

11. Large scale magetic field from local turbulent field

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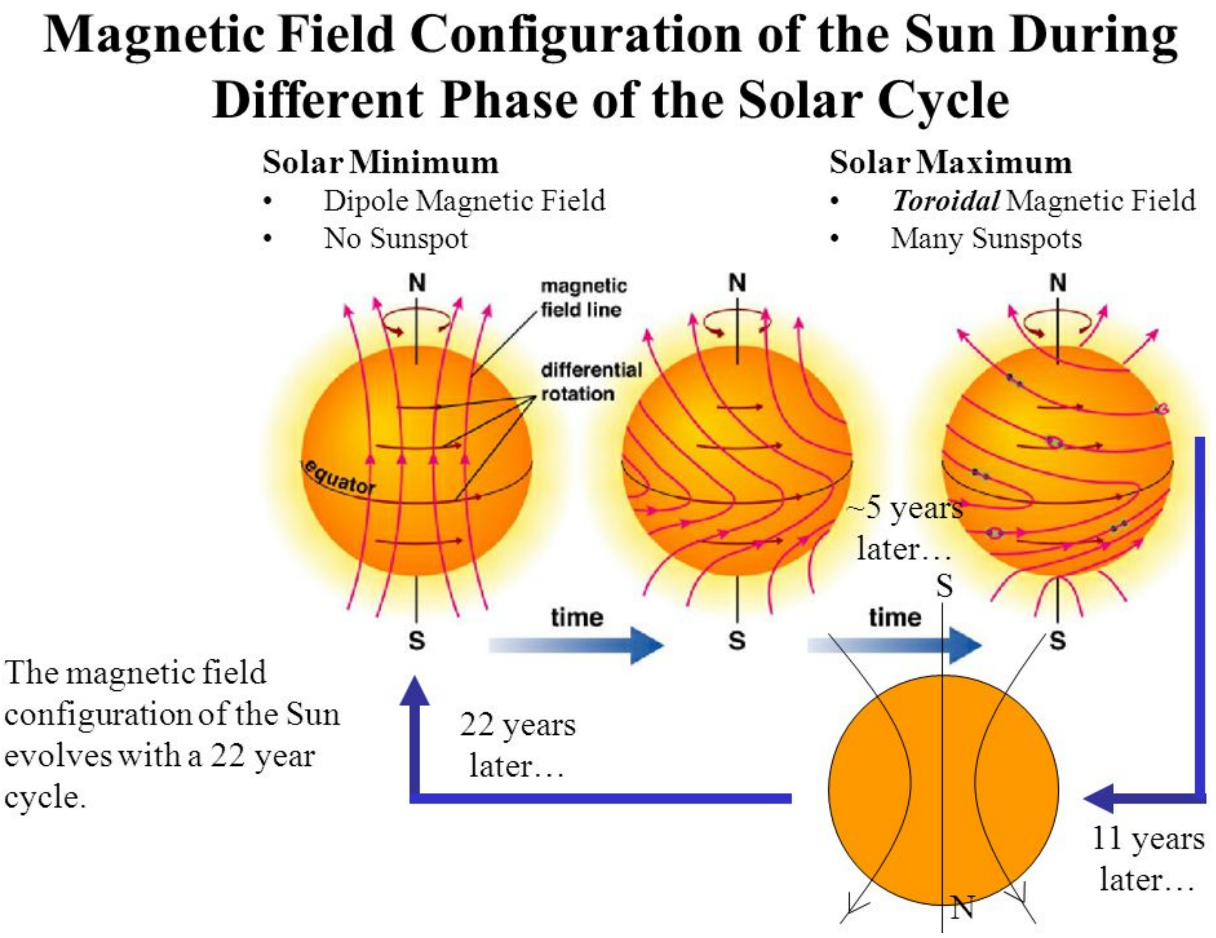
I understand that the key element of the theory is generation of strong (turbulent) magnetic field deeper inside the Sun, and this strong field (B of order 1000 G) shows up only in the Sun spots.

Then, somehow, the small fraction of the field gets organized and a large scale magnetic field (B of order of 1 G) emerges.

This suggests that indeed even a spontaneous generation of large scale field is possible, but it has to be of much lower energy density than the plasma which generated it...



12. Summary

- Magnetic field is of key importance in accretion processes
- It’s role is not yet fully understood in detail
- There is still a gap between observational expectations and the current results of the numerical simulations
- MHD do not yet reproduce the energy spectra, the power spectra, the state transitions and global evolution but the progress is considerable
- GR R 3-D MHD are enormously time-consuming (super-computers are needed)
- The results are sometimes apparently contradictory since they may be based on different initial conditions, boundary conditions or even a computational grid, you have to check always what actually has been assumed in a specific set of computations

NO HOMEWORK