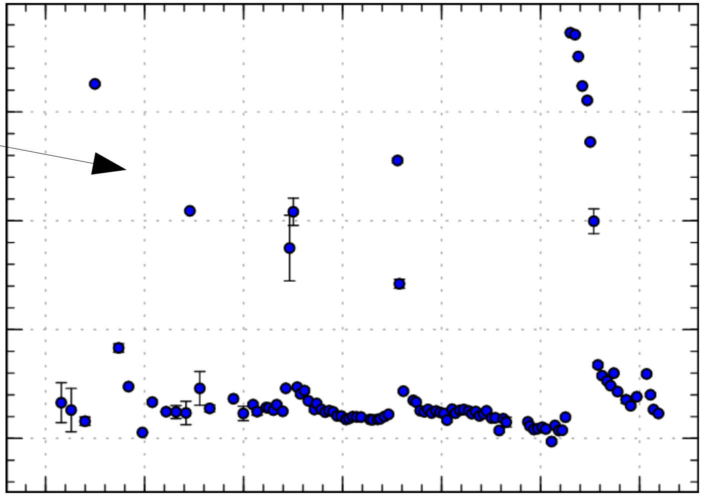
Time evolution of accretion disks, stationarity, stability

1. Introduction

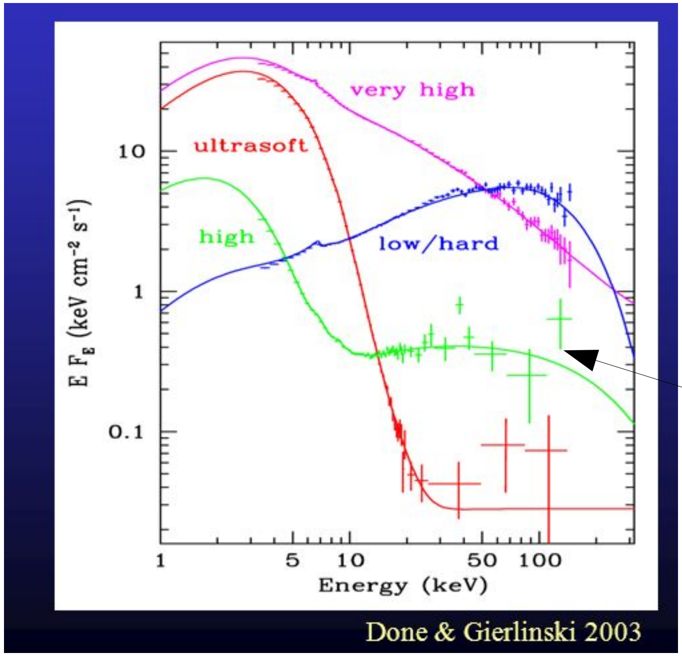
Accretion disks do not have to be stationary. The time evolution can be forced by:

- The initial non-stationary setup of the material
- The change of the outer boundary conditions (modified inflow rate)
- The intrinsic instabilities

We had already one example of the strong evolutionary effects during the previous lectures: dwarf novae outbursts



AAVSO lightcurve of Z Cha (Kato et al. 2014)



Done & Gierlinski 2003

I also discussed in the previous lecture that galactic black hole sources can be in their soft or hard states:

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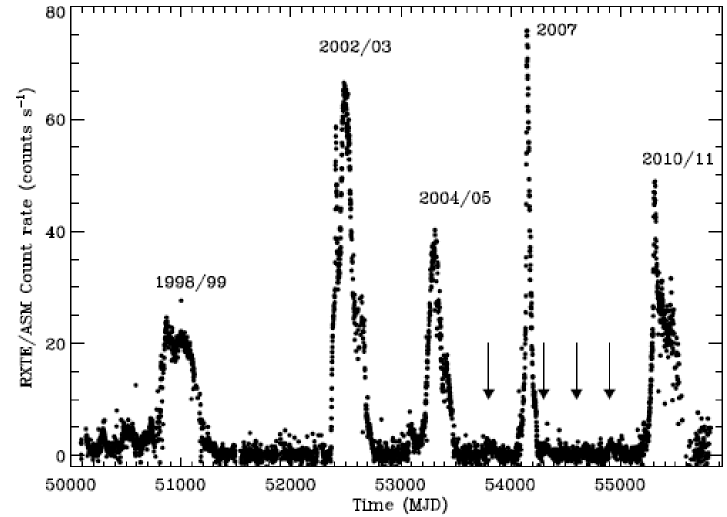
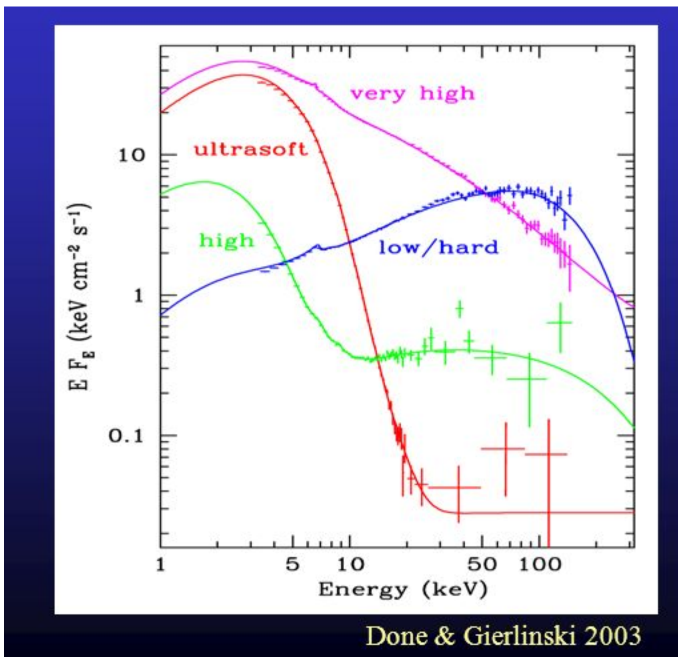
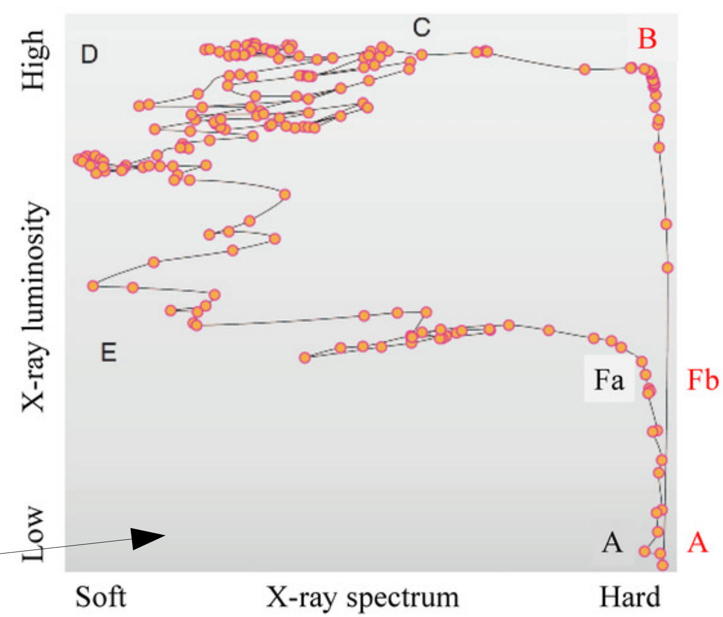


Figure 2. *RXTE*/ASM soft X-ray (1.5-12 keV) light-curve of GX 339−4 for all RXTE lifetime (January 1996 to December

I also discussed in the previous lecture that galactic black hole sources can be in their soft or hard states:



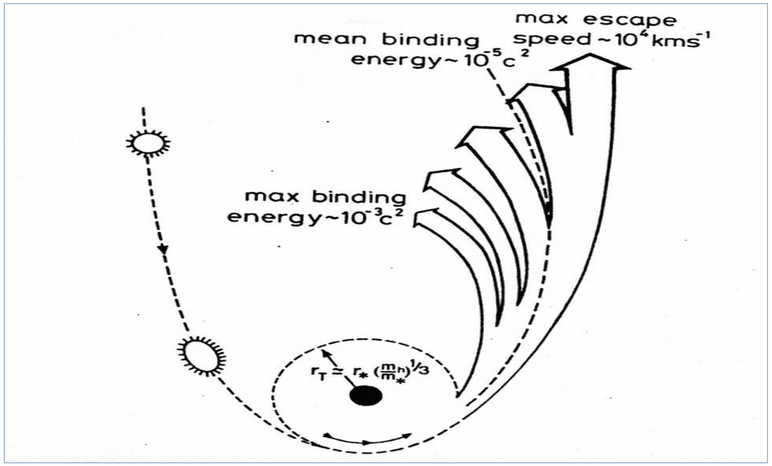
GX 339-4 historical lightcurve from Corbel et al. (2013)

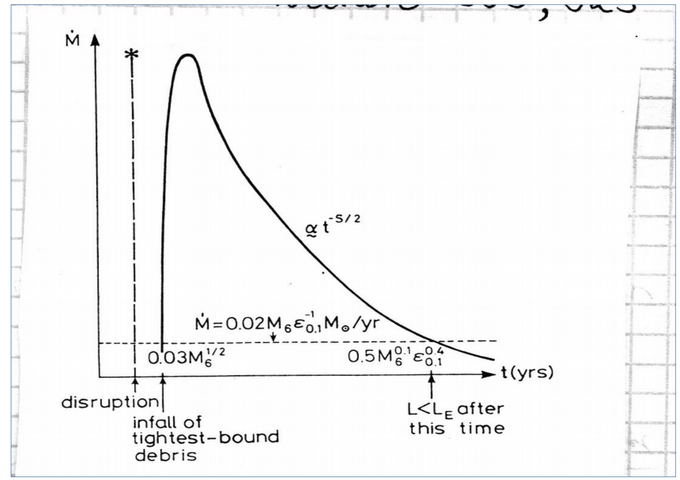


Outburst of the source GX 339-4 in 2002-2003, from Contopoulous 2019, after Fender (2012)

1. Introduction

Rapid changes of luminosity are also seen sometimed from the nuclei of otherwise non-active galaxies. This phenomenon is caused by the disruption of a passing star by the central black hole. The phenomenon lasts for about a 100 days. I mentioned this phenomenon during lecture 3.





theory

$$\rho_{star} < \frac{M}{R^3}$$

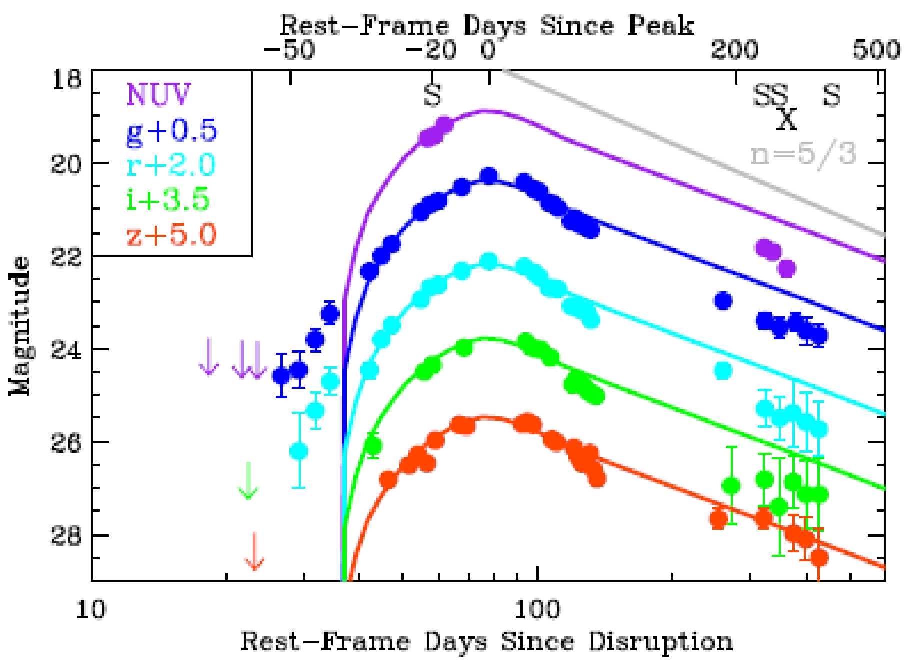


Figure 2
Ultraviolet-optical light curve. The *GALEX NUV* and PS1 g_{P1}-, r_{P1}-, i_{P1}-, and z_{P1}-band light curves of PS1-10jh (with the host galaxy flux removed), plotted against logarithmic

An example from Gezari et al. (2012) of a brightening of otherwise non-active galaxy at z = 0.1696.

2. Timescales in Keplerian disks

If we see some outbursts in a given object we would like to have a clue to estimate what might have happened. For example, we see outbursts like this (*Miniutti et al. 2019*).

The source of radiation is low mass AGN (black hole mass 4 x 10^5 M_s) at low redshift (0.018).

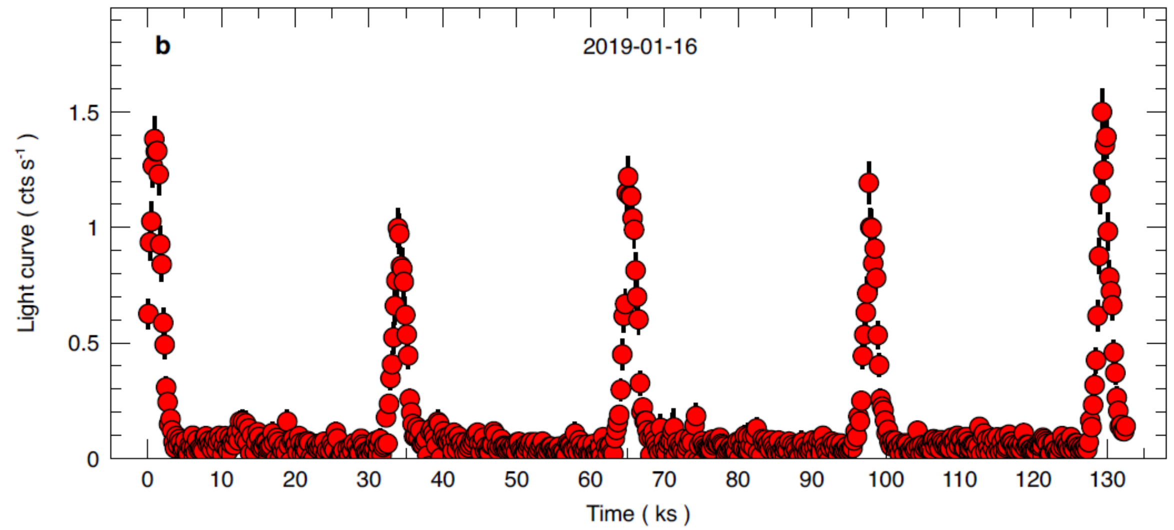


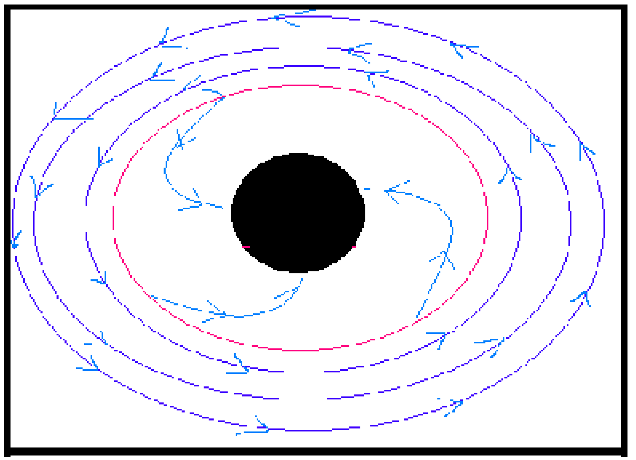
Figure 1 | X-ray QPEs in XMM-Newton and Chandra observations, 2018 December onwards

What is happening?

The basic help comes from the knowledge of the expected timescales in Keplerian disks, as that allows to estimate **if** the process is related to a phenomenon in Keplerian disk, and if so, what is the likely underlying mechanism.

2.1 Dynamical timescale

There are two timescales with are related to the dynamical motion in the Keplerian disk.



First one is the orbital timescale. At a radius r, the angular velocity of the keplerian disk is:

$$\Omega_K = \sqrt{\frac{GM}{r^3}}$$

The period of the orbital motion is

$$t_{dyn} = \frac{1}{\Omega_K}$$

2.1 Dynamical timescale

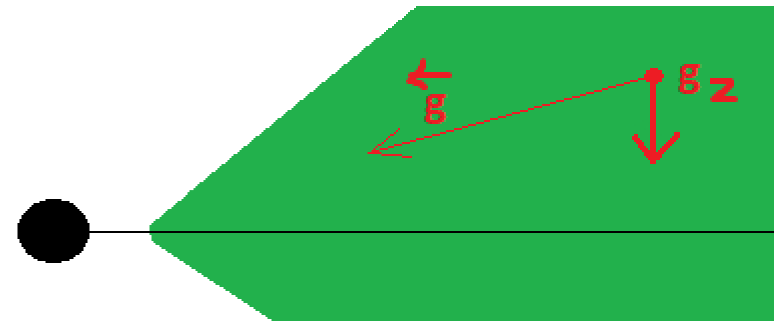
The second dynamical timescale is related to the question how fast the Keplerian disk can restore the hydrostatic equilibrium.

We has a time-dependent version of the equation of motion in 'z' direction in lecture 7

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) \quad = \quad -\frac{\partial P}{\partial z}+\left(\frac{1}{r}\frac{\partial(r\tilde{\tau}_{rz})}{\partial r}+\frac{1}{r}\frac{\partial \tilde{\tau}_{\theta z}}{\partial \theta}+\frac{\partial \tilde{\tau}_{zz}}{\partial z}\right)+\rho g_z$$

And we then derived the hydrostatic balance eqution in a

$$\frac{1}{\rho}\frac{dP}{dz} = -\frac{GMz}{r^3} \qquad \textit{hydrostatic equilibrium}$$



We also derived the vertically-averaged equation $\frac{v_s}{v_\phi}=\frac{H}{r}$

We can now derive in a similar way the timescale related to the vertical motion just by saying that now the term (1) on the left and term (2) on the right (and also the last term on the right) are comparable:

$$\rho\frac{v_z}{t_z}=\frac{P}{H} \qquad v_z=\frac{H}{t_z} \quad \Rightarrow \quad \rho\frac{H}{t_z^{\ 2}}=\frac{P}{H} \quad \Rightarrow \quad t_z^{\ 2}=\frac{\rho}{P}H^2$$

We introduced sound speed before $v_s=\frac{P}{\rho}$ and using this concept we can express the timescale needed to restore the hydrostatic equilibrium as

$$t_z=\frac{H}{v_s}$$

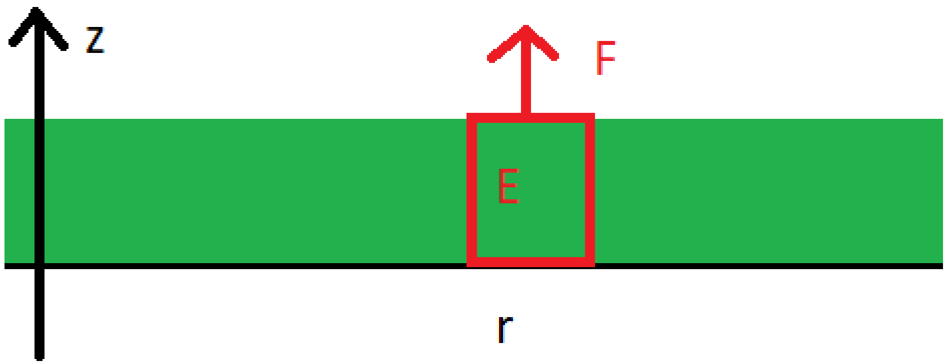
This timescale is equal to time needed by the sound waves to propagate in the vertical direction.

$$t_z=t_{dyn}=\frac{1}{\Omega_K}$$

2.2 Thermal timescale

We can estimate the thermal timescale also by looking at the time-dependent version of equations but we can actually guess what to do.
Like in stars, the thermal timescale (at a given disk radius) can be introduced as the ratio of the energy content to the radiation flux.

$$t_{th} = \frac{E}{F}$$



The energy density is proportional to the pressure for a perfect fluid, so

$$E = P\,H$$

We can use the prescription for the flux $F_{rad}(r) = \frac{3GM\dot{M}}{8\pi r^3}\left(1-\sqrt{\frac{3}{x}}\right)$ but it will not lead to an expression in the interesting form.
So instead we will refer to the equation of the energy generation in the disk $\frac{F_{rad}}{H} = \frac{3}{2}\alpha P \Omega_K$

$$t_{th} = \frac{P\,H}{\alpha P \Omega_K H}$$

Here I dropped the factor 2/3 (it is an estimate anyway).

So finally we get a very simple and very important formula

$$t_{th} = \frac{1}{\alpha \Omega_K} = \frac{1}{\alpha} t_{dyn}$$

The thermal timescale is longer just by a factor given by the viscosity parameter!

2.3 Viscous timescale

In analogy, it is easy to guess a simple estimate of the timescale for an inflow of the material in the Keplerian disk

$$t_{visc} = \frac{r}{v_r}$$

It is the time needed to cover the distance r to the blacl hole with a speed v_r determined at the radius r under discussion

We have to start from the continuity equation for a disk to get v_r, simplyfy it to get

$$\dot{M}(t,r) = -\int_{-\infty}^{+\infty} 2\pi r \rho v_r dz$$

$$\dot{M} = 2\pi r \rho v_r H$$

and then we have to combine this with the equations.

$$\frac{1}{\rho}\frac{P}{H} = \frac{GMH}{r^3}$$

$$\frac{F_{rad}}{H} = \frac{3}{2}\alpha P \Omega_K$$

$$F_{rad}(r) = \frac{3GM\dot{M}}{8\pi r^3}\left(1-\sqrt{\frac{3}{x}}\right)$$

Finally, we drop the factor related to the inner boundary and the remaining numerical factor finally to get

$$t_{visc} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 \frac{1}{\Omega_K} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 t_{dyn}$$

2.4 Timescale summary

$$t_z = t_{dyn} = \frac{1}{\Omega_K}$$

Dynamical timescale – the shortest one – corresponds to the orbital period but also describes the propagation of sound waves and time needed to restore the hydrostatic equilibrium.

$$t_{th} = \frac{1}{\alpha\,\Omega_K} = \frac{1}{\alpha} t_{dyn}$$

The thermal timescale desribed the time needed to restore the thermal equilibrium, i.e. the balance between heating and cooling. It depends only on the viscosity parameter α

$$t_{visc} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 \frac{1}{\Omega_K} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 t_{dyn}$$

The viscous timescale corresponds to the timescale needed for significant redistribution of the mass inside the disk. It depends both on the viscosity parameter α but also strongly depends on the disk thickness ratio (H/r).

In Keplerian disk $t_{visc} \gg t_{th} \gg t_{dyn}$

In further discussion we will assume that the disk is in the hydrostatic equilibrium (sound waves, dynamical disk pulsations neglected) and we will study some aspects of slow viscous and thermal evolution.

3. Viscous evolution of a ring and accretion disk formation

Why we start with that?

- Because, under some assumptions, there is a semi-analytical solution to the problem
- It is indeed interesting in the context of tidal disrution events (TDE)

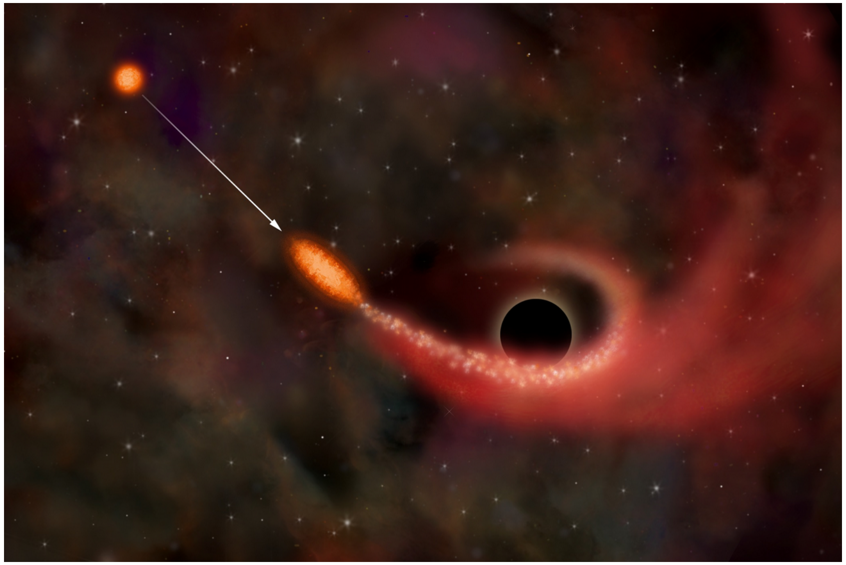


Illustration of TDE from AAS Nova

We cannot get analytically a whole event like that, but we can have a semi-analytical solution starting from a narrow ring.

First we have to go back to two time-dependent equations: continuity equation

and angular momentum transfer equation (lecture 6)

$$4\pi r\frac{\partial \Sigma}{\partial t} - \frac{\partial \dot{M}}{\partial r} = 0$$

$$\dot{M}\frac{\partial l_K}{\partial r}=\frac{\partial \Theta}{\partial r}$$

We now average the equation to get the result

$$\Theta=\int_{-\infty}^{\infty} 2\pi r^2 t_{r\phi}\, dz$$

$$\Theta=2\pi r^2 t_{r\phi} H$$

and change the parametrization of the viscosus torque

$$t_{r\phi}=-r\frac{\partial \Omega_K}{\partial r}\nu\rho$$

$$t_{r\phi}=\alpha P$$

This is a different parametrization than Shakura-Sunyaev torque

3. Viscous evolution of a ring and accretion disk formation

$$4\pi r \frac{\partial \Sigma}{\partial t} - \frac{\partial \dot{M}}{\partial r} = 0$$

$$\dot{M} \frac{\partial l_K}{\partial r} = \frac{\partial \Theta}{\partial r}$$

$$t_{r\phi} = -r \frac{\partial \Omega_K}{\partial r} \nu \rho$$

This is a different parametrization than Shakura-Sunyaev torque

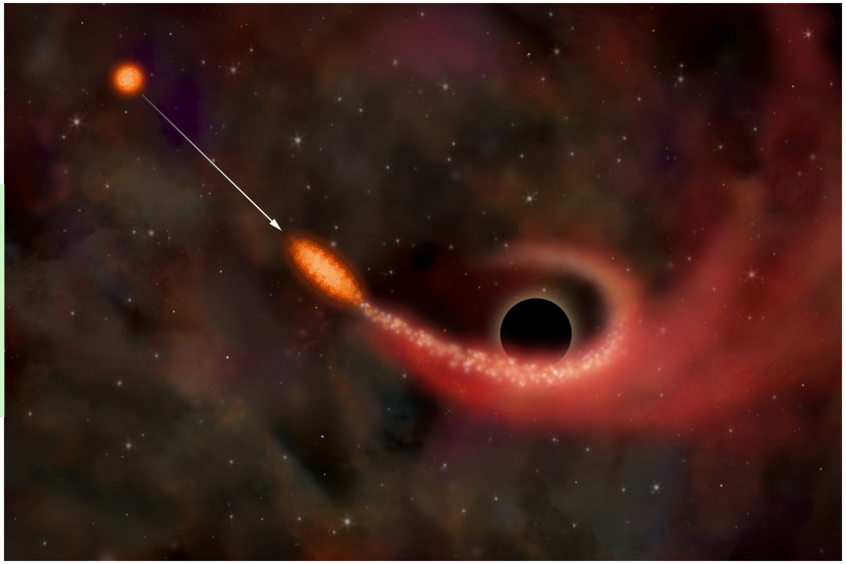


Illustration of TDE from AAS Nova

But it helps to get a nice result

$$4\pi r \frac{\partial \Sigma}{\partial t} = \frac{\partial \dot{M}}{\partial r} = \frac{\partial}{\partial r}\left[\frac{1}{\frac{dl_K}{dr}} \frac{\partial}{\partial r}\left(r \frac{d\Omega_K}{dr} \nu \rho 4\pi r^2 H\right)\right]$$

$$\Sigma = \rho H$$

We now assume that the kinematic viscosity is CONSTANT. This is why we changed parametrization. This simplifies our job since we do not have now to consider the thermal balance. We have one differential equation for the time evolution of the disk surface density

$$\frac{\partial \Sigma}{\partial t} = \frac{3\nu}{r} \frac{\partial}{\partial r}\left[r^{1/2} \frac{\partial}{\partial r}\left(\Sigma r^{1/2}\right)\right]$$

3. Viscous evolution of a ring and accretion disk formation

$$\frac{\partial \Sigma}{\partial t} = \frac{3\nu}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\Sigma r^{1/2}\right)\right]$$

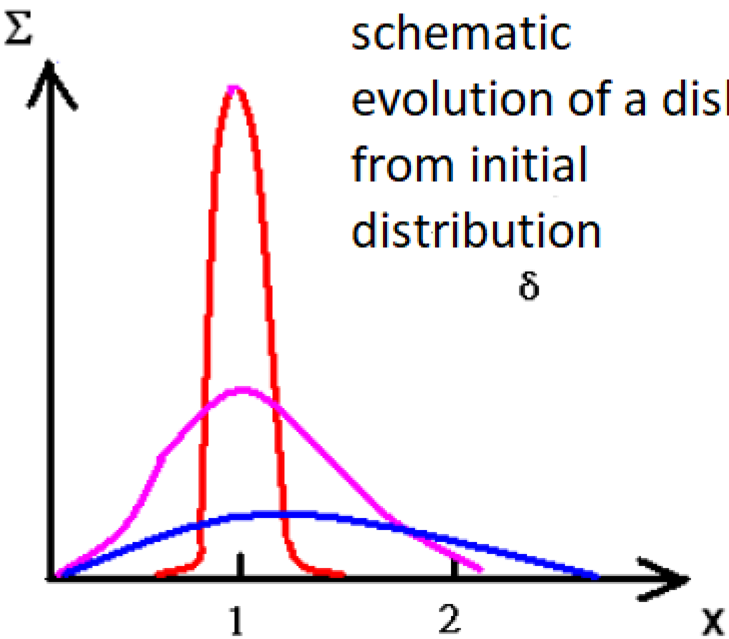
If the initial mass distribution is described by Dirac delta function δ

$$\Sigma(r, t=0) = \frac{m}{2\pi r_0}\delta(r - r_0)$$

Then the evolution is described in dimensionless units by

$$\Sigma(x,\tau) = \frac{m}{\pi R_o^2}\tau^{-1}x^{-1/4}\exp\left(-\frac{1+x^2}{\tau}\right)I_{1/4}(2x/\tau)$$

Here $\tau = 12\nu t / r_0^2$ and x = r/r_0. I is the Bessel function.



The expression for the accretion rate through the inner disk radius can be found even in a more general case

$$\nu = \nu_0\left(\frac{R}{R_0}\right)^n$$

$$\dot{M}(t) = \frac{M_0}{t_{\rm visc}}\frac{(2\mu^2)^\mu}{\Gamma(\mu)}\tau^{-1-\mu}\exp\left(-\frac{2\mu^2}{\tau}\right),$$

Here $\mu = \frac{1}{4-2n}$; so the most standard solution n = 0 corresponds to μ = ¼, and the slope of the power law decay phase is thus -5/4, the classical derivation gives -5/3.

Expressions from Zdziarski (2009)

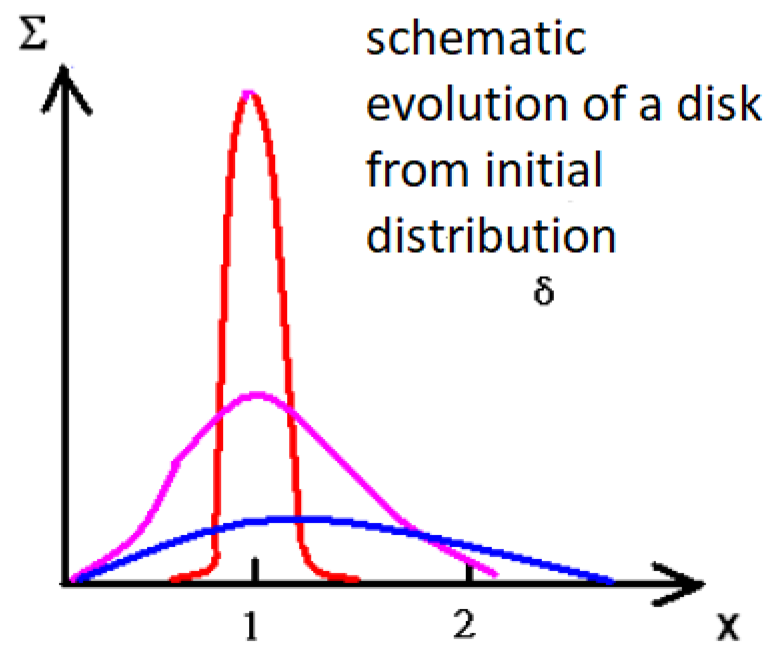
3. Viscous evolution of a ring and accretion disk formation

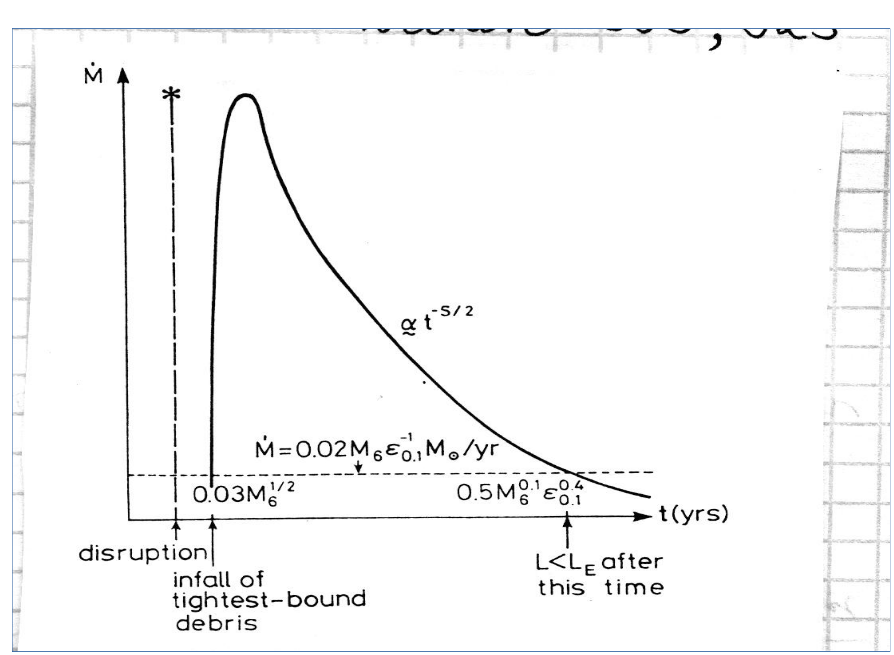
$$\frac{\partial \Sigma}{\partial t}=\frac{3\nu}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\Sigma r^{1/2}\right)\right]$$

If the initial mass distribution is described by Dirac delta function δ

$$\Sigma(r,t=0)=\frac{m}{2\pi r_0}\delta(r-r_0)$$

- The disk spreads in both directions from the initial ring position
- The accretion rate through the inner edge initially rises rapidly, and then the decay has a power law character





$$\dot{M}(t)=\frac{M_0}{t_{\rm visc}}\frac{(2\mu^2)^{\mu}}{\Gamma(\mu)}\tau^{-1-\mu}\exp\left(-\frac{2\mu^2}{\tau}\right),$$

4. Thermal and viscous stability of radiation pressure dominated Keplerian disk

Calendar:
1973 – Shakura & Sunyaev (1973) αP disk models
1973 - Lightman & Eardley (1973) – those disks are viscously unstable
1974 - Pringle, Reen & Pacholczyk (1974) – those disks are thermally unstable

That discouraged some people from using disk SS models despite their possibility to fit the data for many objects/states. Others tried to modify the prescription. Some others still used the models assuming that the disks can handle the issue themselves one way or another. For example, many unstable stars do pulsations but nothing more than tha, they are not disrupted.

The key problem (lecture 7): inner region of SS

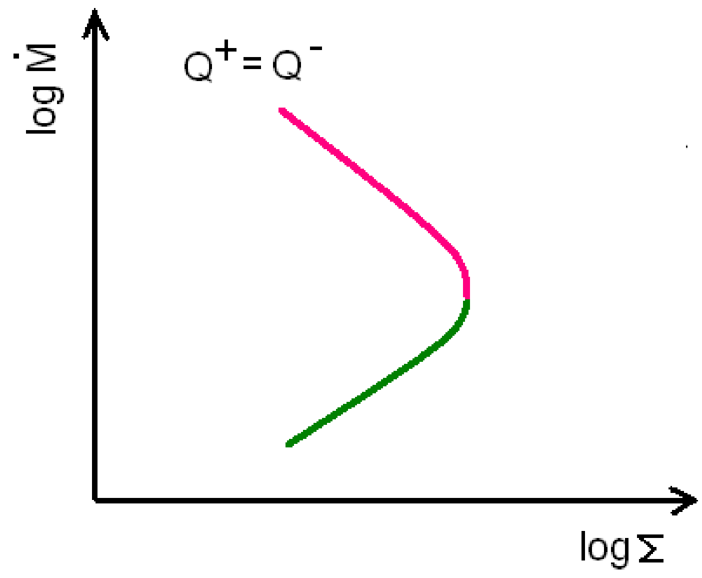
$$u_0 \left[\frac{\mathrm{g}}{\mathrm{cm}^2}\right] = \frac{64\pi}{9\alpha} \frac{c^2}{\sigma^2} \frac{1}{\omega \dot{M}(1 - r^{-1/2})}$$
$$= 4.6\alpha^{-1} \dot{m}^{-1} r^{3/2} (1 - r^{-1/2})^{-1},$$

Middle region of SS

$$u_0 = 1.7 \cdot 10^5 \alpha^{-4/5} \dot{m}^{3/5} m^{1/5} r^{-3/5} (1 - r^{-1/2})^{3/5}$$

So in general, at a given radius, the plot of the surface density as a function of accretion rate has the following shape

Surface density (ρH)



4. Thermal and viscous stability of radiation pressure dominated Keplerian disk

Let us first assume that we study the radiation pressure dominated branch in thermal timescale. So we assume the hydrostatic equilibrium (that is faster), and we assume that surface density of the disk does not change (that is slow, no time to rearange the mass in thermal timescale). But we do not assume the thermal balance

$$\frac{P}{\rho H} = \frac{G M H}{R^3}$$ Hydrostatic equilibrium

$$Q^+ = \frac{3}{2}\alpha P \Omega_K H$$ Heating

$$Q^- = \frac{4\sigma T^4}{3\kappa_{es}\rho H}$$ Cooling

$$P = \frac{1}{3} a T^4$$

Radiation pressure dominates

If we use the first equation and the expression for the pressure, and assume $\Sigma = \rho H$ constant we will get the heating and cooling as functions of the temperature

$$Q^+ \propto T^8 \qquad Q^- \propto T^4$$

$$\Rightarrow \quad \frac{d\ln Q^+}{d\ln T} = 8 \qquad \frac{d\ln Q^-}{d\ln T} = 4 \quad \Rightarrow \quad \frac{d\ln Q^+}{d\ln T} > \frac{d\ln Q^-}{d\ln T}$$

If the temperature due to some fluctuation is too high for a heating/cooling balance, heating will become larger than cooling. This means instability. The temperature will rise continuously.

4. Thermal and viscous stability of radiation pressure dominated Keplerian disk

If we redo the same for the gas dominated branch, the result will be different:

$$\frac{P}{\rho H} = \frac{G M H}{R^3}$$

Hydrostatic equilibrium

$$P \propto \rho T$$

$$Q^+ = \frac{3}{2} \alpha P \Omega_K H$$

Heating

Gas pressure dominates

$$Q^- = \frac{4 \sigma T^4}{3 \kappa_{es} \rho H}$$

Cooling

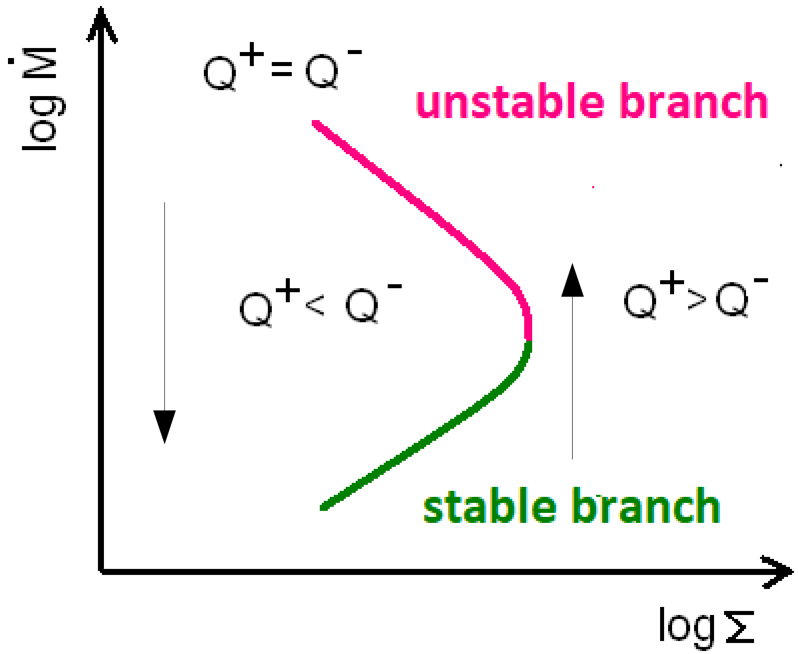
If we use the first equation and the expression for the pressure, and assume $\Sigma = \rho H$ constant we will get the heating and cooling as functions of the temperature

$$Q^+ \propto T \qquad Q^- \propto T^4$$

$$\Longrightarrow \quad \frac{d\ln Q^+}{d\ln T} = 1 \qquad \frac{d\ln Q^-}{d\ln T} = 4 \quad \Longrightarrow \quad \frac{d\ln Q^+}{d\ln T} < \frac{d\ln Q^-}{d\ln T}$$

If the temperature due to some fluctuation is too high for a heating/cooling balance, cooling will become larger than cooling, and the balance will be restored.

4. Thermal and viscous stability of radiation pressure dominated Keplerian disk



It is more complicated to repeat the argument of Lightman & Eardley for viscous evolution (we then assume the thermal balance) but the result is

$$\frac{d \log \dot{M}}{d \log \Sigma} > 0 \qquad \textit{stable solution}$$

$$\frac{d \log \dot{M}}{d \log \Sigma} < 0 \qquad \textit{unstable solution}$$

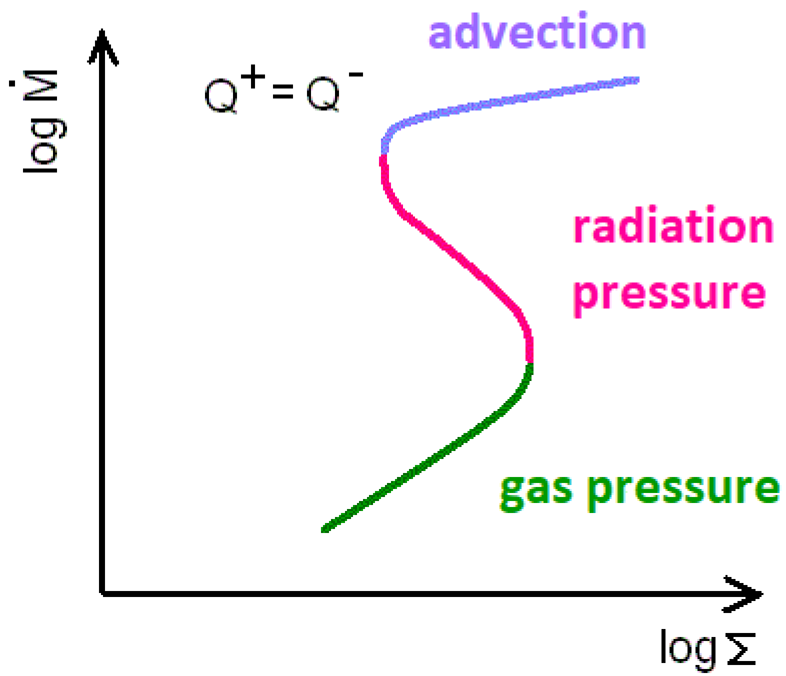
Both thermal and viscous instability work at the same parameter range (Shakura & Sunyaev 1976). The problem of what happens at high accretion rate is solved if we allow for the advection term which was shortly discussed in the context of ISCO.

Advection describes the thermal energy which is not emitted locally from the disk surface but is contained in the flow

$$F_{adv} \approx \rho v_r v_s^2 H \qquad \frac{F_{adv}}{F} \approx \left(\frac{H}{R}\right)^2 \frac{1}{1-\sqrt{R_{ms}/R}}$$

If H/R of order of 1, advection becomes important, but also we have departures from the assumptions of Keplerian disks. Such solutions are know as **slim disks** (Abramowicz, Czerny, Lasota & Szuszkiewicz 1988).

4. Slim disks – closing the system



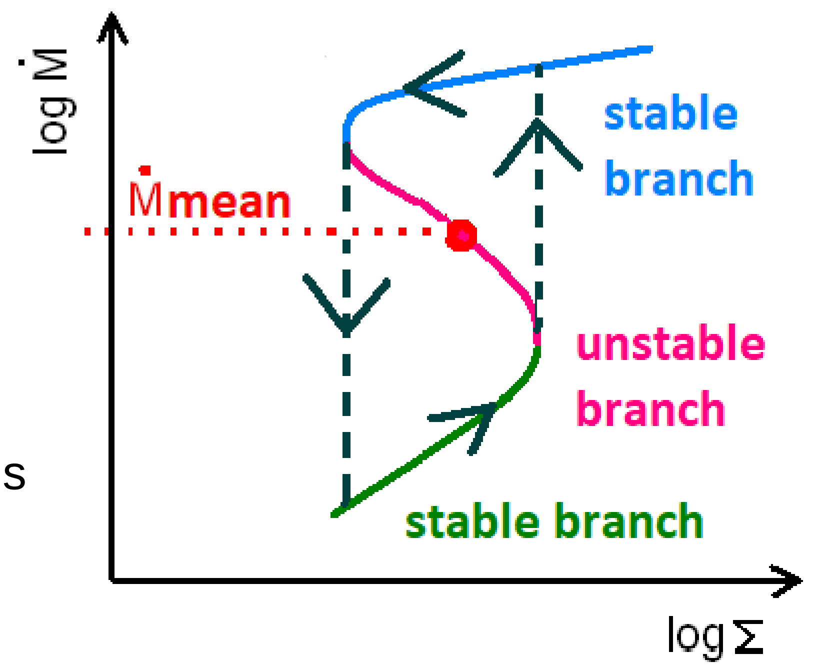
New branch is again stable, since the tole of cooling is taken over by advection

$$Q^{-} \propto T^{12}$$

And now wins over the heating.

So we have now a stable upper branch and a stable lower branch, but still there is a rangle of accretion rates which give unstable solution for this specific radius.

What happens if we choose the value of accretion rate which at this radius corresponds to the unstable solution?

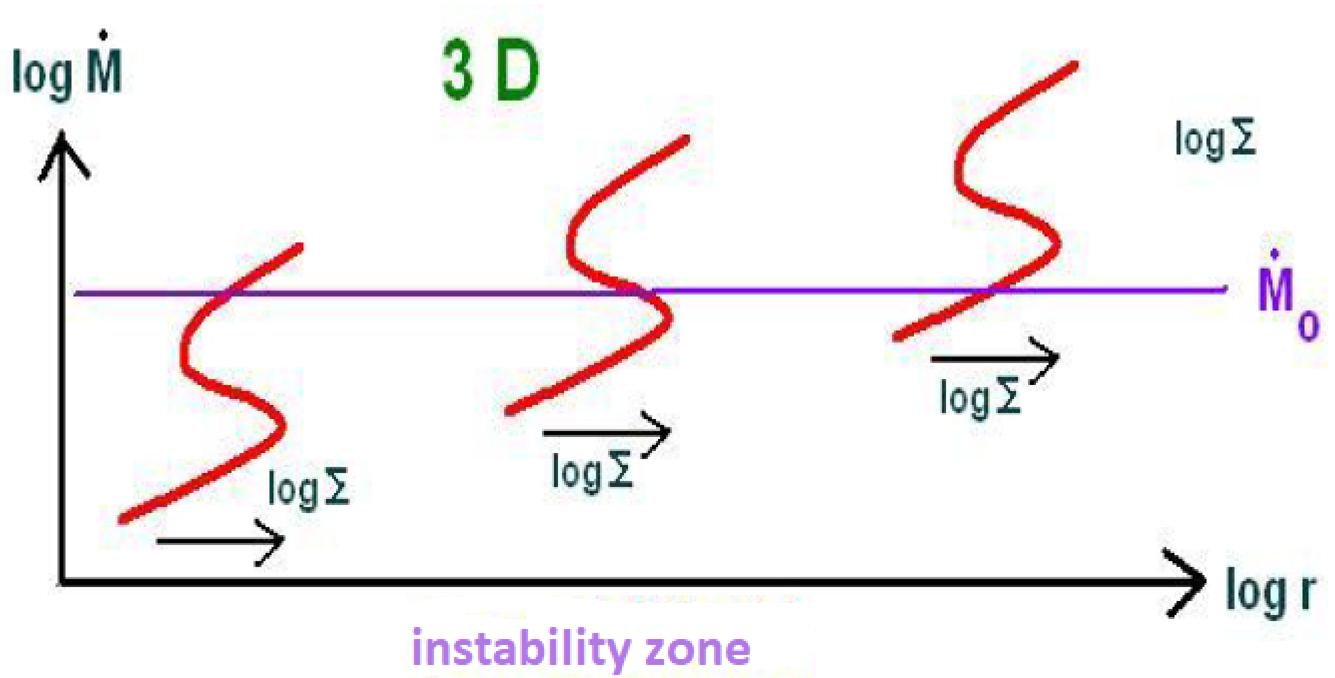


We will get a limit-cycle behaviour:

Lower and upper part of the evolution are in the viscous timescale, while the verstical parts are in thermal timescale, under constant surface density (not quite true for the upper branch...)

5. Slim disks – are they stable?

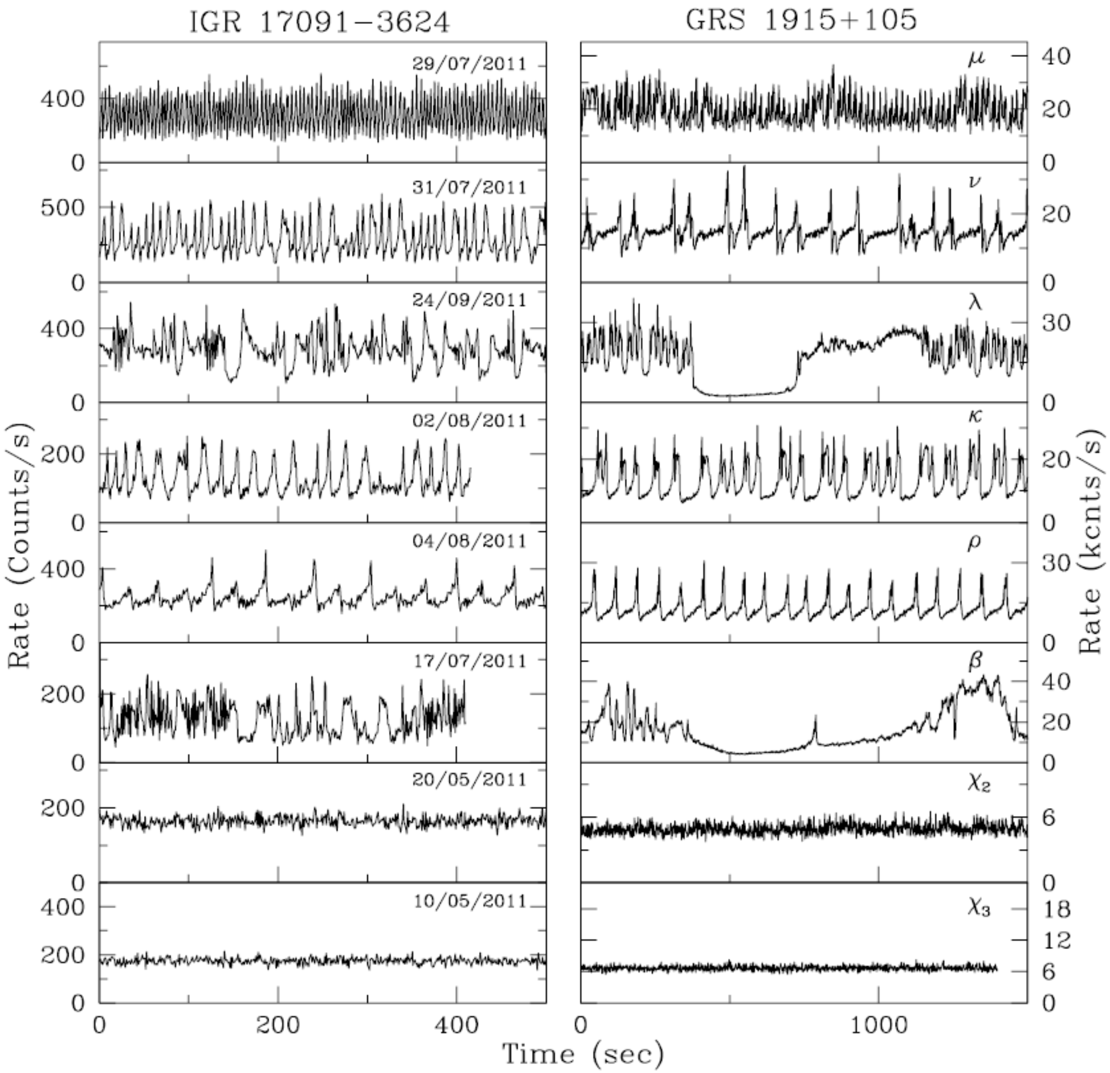
Studying the accretion disk behaviour cannot be done just at a single radius since the accretion flow proceeds from the outer radius down to ISCO. If we choose high accretion rate so the inner part of the disk is stable will the whole disk be stable?



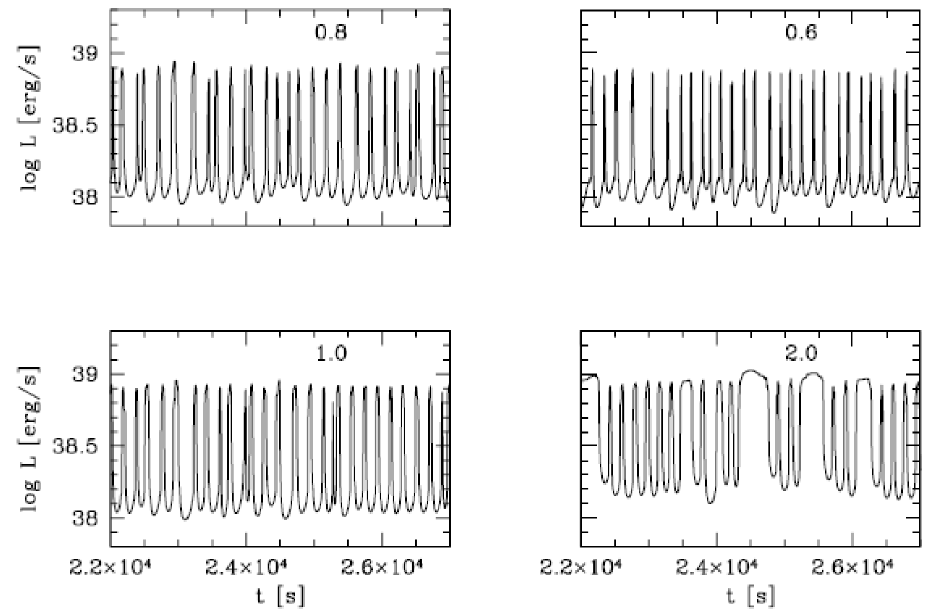
For the same adopted external accretion rate, the outermost part of the disk will be gas-dominated and stable, and the intermediate part will be radiation-pressure dominated and unstable.

Only disks which have low accretion rates and are not radiation-pressure dominated in any part of the disk are stable (from this point of view – but wait a minute!!!)

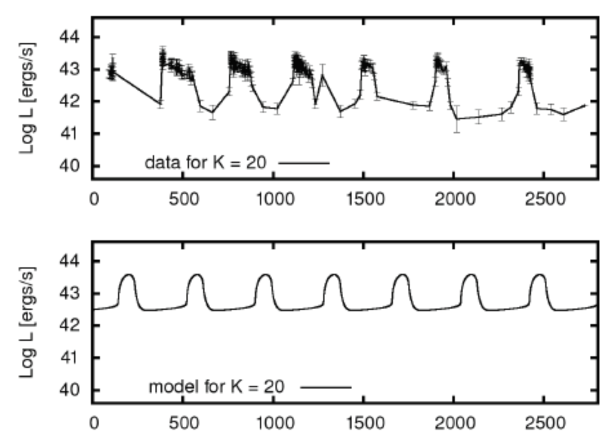
6. Slim disks – time-dependent computations



Data for two galactic sources showing **heartbeat** *states (from Pal & Chakrabarty 2018)*



Models from Janiuk et al. (2000)

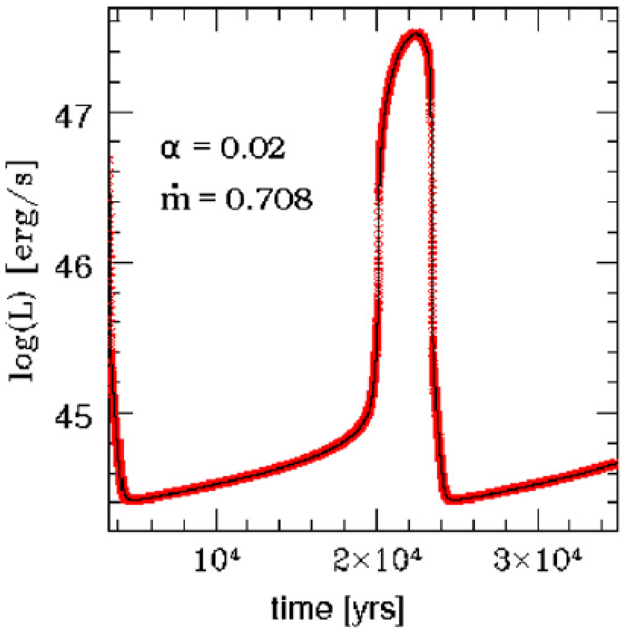


Example of application to Intermediate mass black hole HLX X-1 (Grzedzielski et al. 2017)

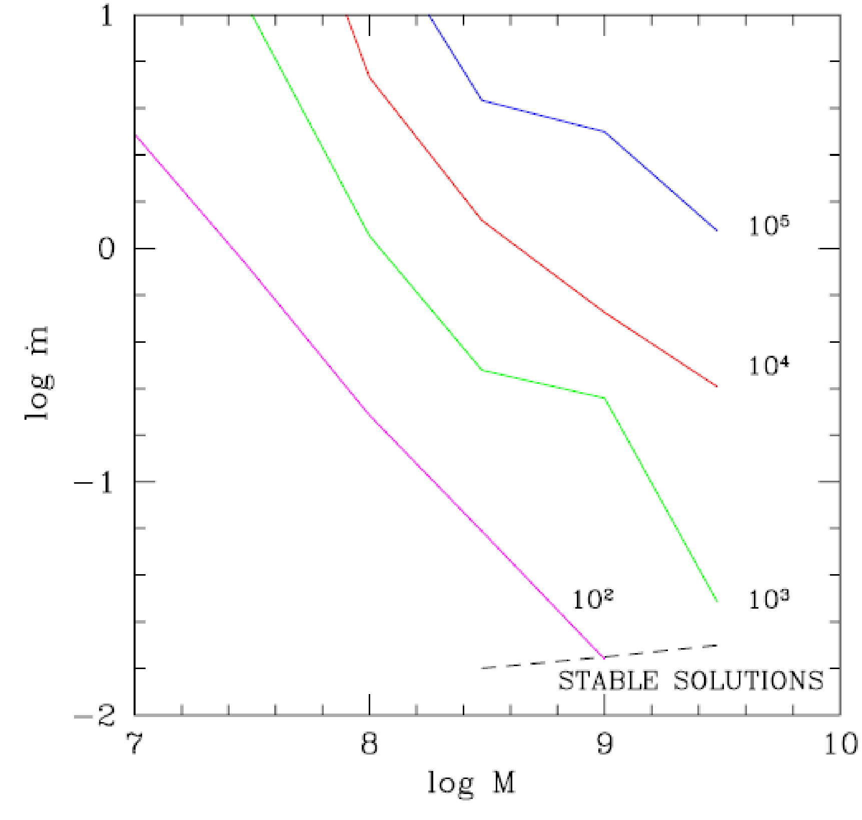
6. Slim disks – time-dependent computations

In the case of AGN the radiation pressure instability should be present but the timescales are long. The Keprian period is of order of months, and the aspect ratio H/r generally small.

$$t_{visc} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 \frac{1}{\Omega_K} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 t_{dyn}$$



Example of a lightcurve from Czerny et al. (2009), the outburst lasts a few thousands years.

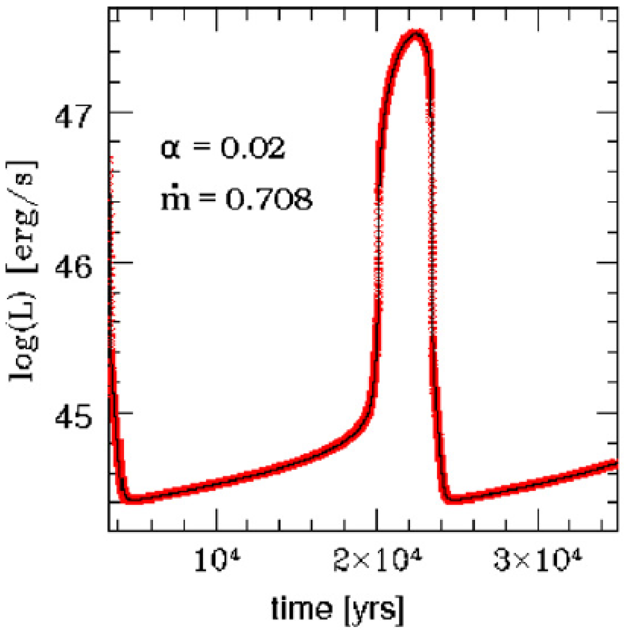


And we postulated that enhanced number of radio young sources actually represents reactivation due to radiation pressure instability (numbers refer to AGN ages in years).

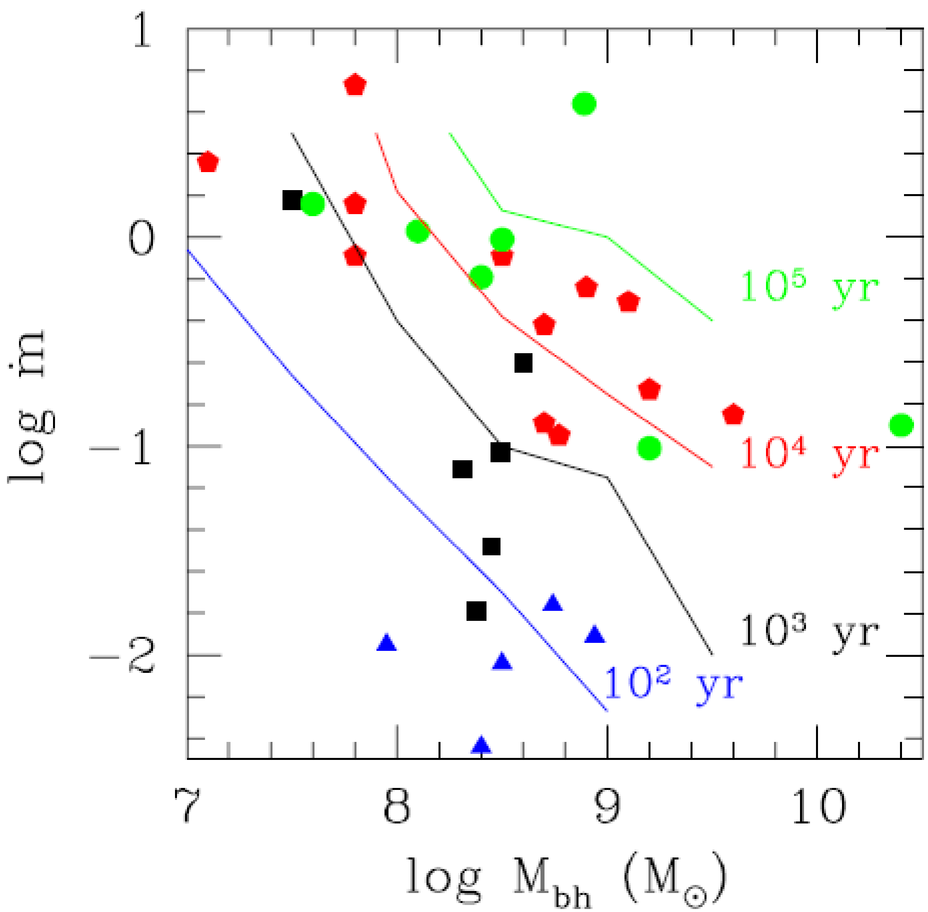
6. Slim disks – time-dependent computations

In the case of AGN the radiation pressure instability should be present but the timescales are long. The Keprian period is of order of months, and the aspect ratio H/r generally small.

$$t_{visc} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 \frac{1}{\Omega_K} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 t_{dyn}$$



Example of a lightcurve from Czerny et al. (2009), the outburst lasts a few thousands years.

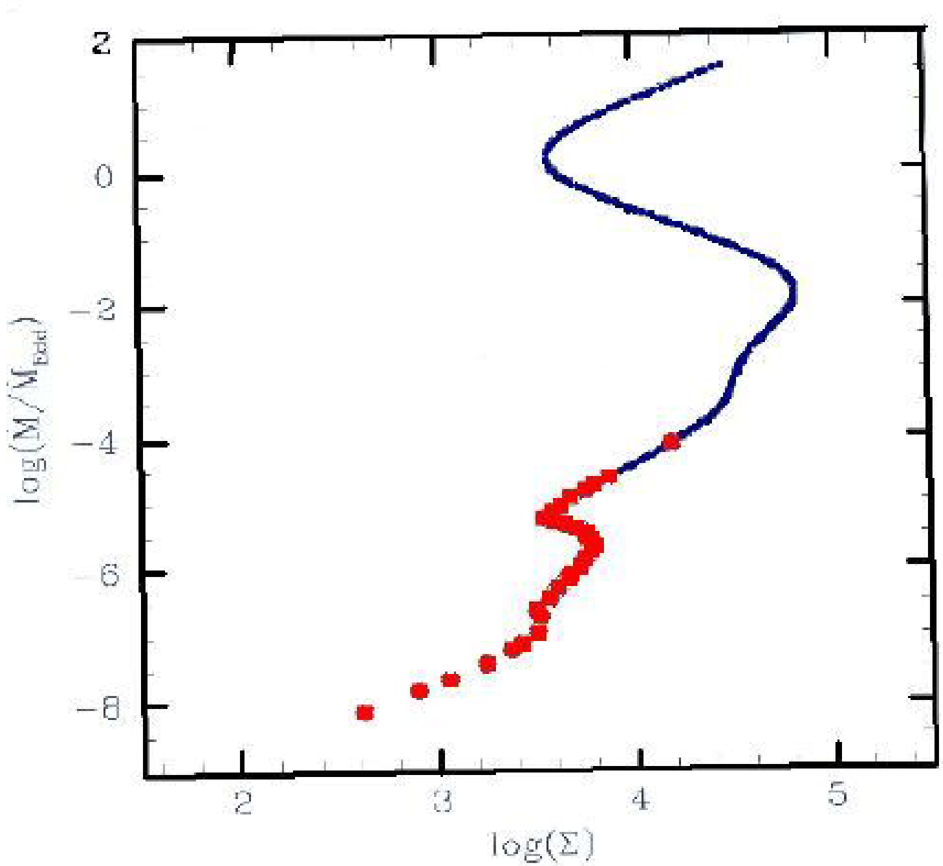


And Wu (2009) was able to determine black holes for our radio sources and locate them on our maps.

Still, it is an open issue of the intermittent AGN activity is indeed related to radiation pressure instability.

7. Ionization instability in gas-dominated Keplerian disks

I mentioned that gas-dominated disks can be also unstable.



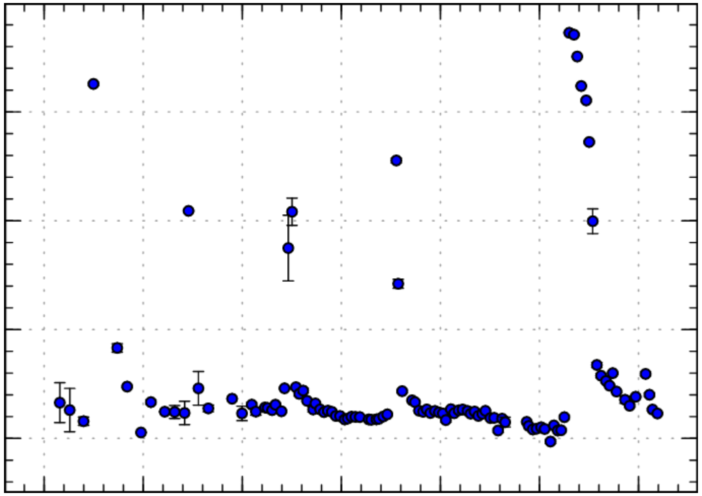
This is a complete equilibrium curve which I calculated a long time ago for NGC 4151, at 10 R_{schw}.

At very low accretion rates (or at very large distances) the partial ionization of hydrogen and helium forms a ‘wiggle’.

It is not clear if this is relevant for AGN, But this was actually noticed first for disks around white dwarfs (cataclysmic variables) by Meyer & Meyer-Hoffmeister (1981) and Smak (1984).

This instability is a generally accepted model of the outbursts of dwarf novae, like that one I showed at the beginning of the lecture.

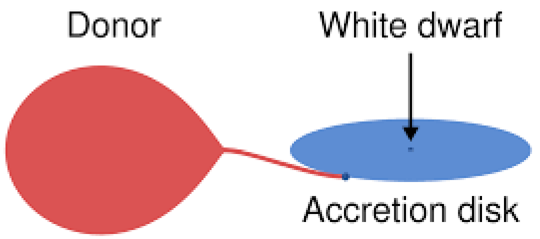
Outbursts are semi-regular, last usually for days, and repeat after weeks/years.



AAVSO lightcurve of Z Cha (Kato et al. 2014)

7. Ionization instability in gas-dominated Keplerian disks

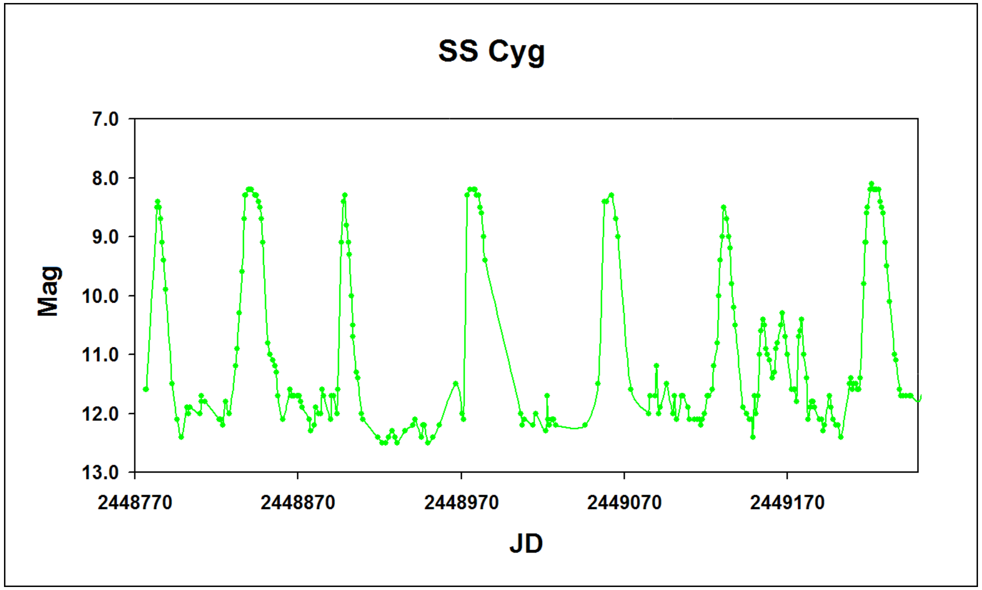
I mentioned that gas-dominated disks can be also unstable.



But this was actually noticed first for disks around white dwarfs (cataclysmic variables) by Meyer & Meyer-Hoffmeister (1981) and Smak (1984).

Outbursts are semi-regular, last usually for days, and repeat after weeks/years.

This instability and subsequent outbursts are thus happening in the disk, and are not caused by the variable transfer from the companion.



Aspects still under discussion:
- Superoutbursts
- Superhumps
- disk state between outbursts

SS Cyg lightcurve from http://rjm-astro.net/sscyg.html

7. Ionization instability in gas-dominated Keplerian disks

The same instability works in galactic sources containing black holes, and it is responsible for the X-ray-novae phenomenon. I mentioned the outburst of this source during lecture 8.

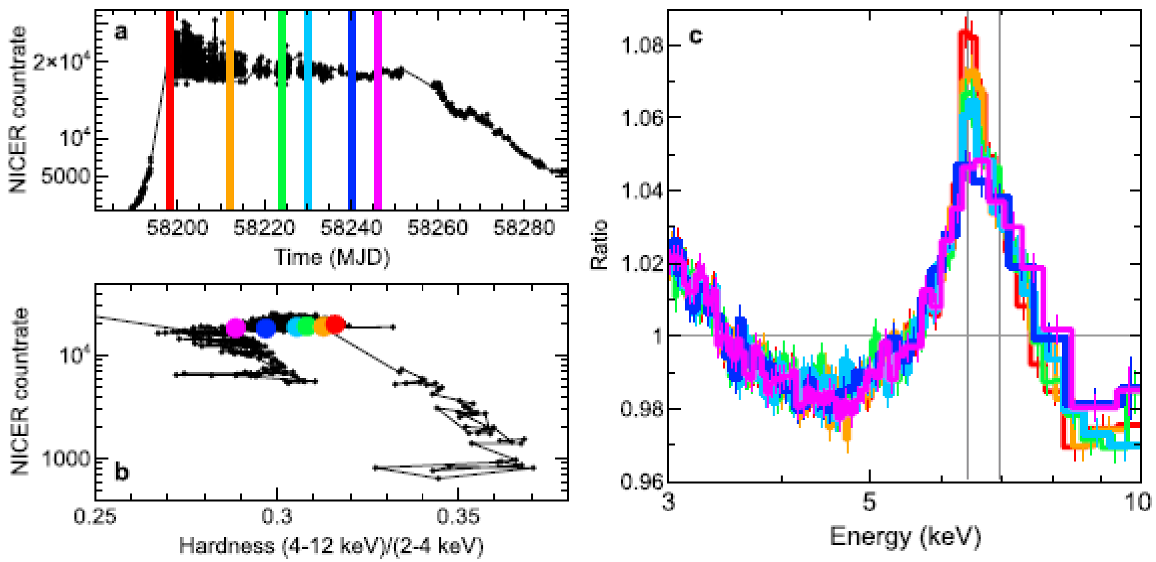
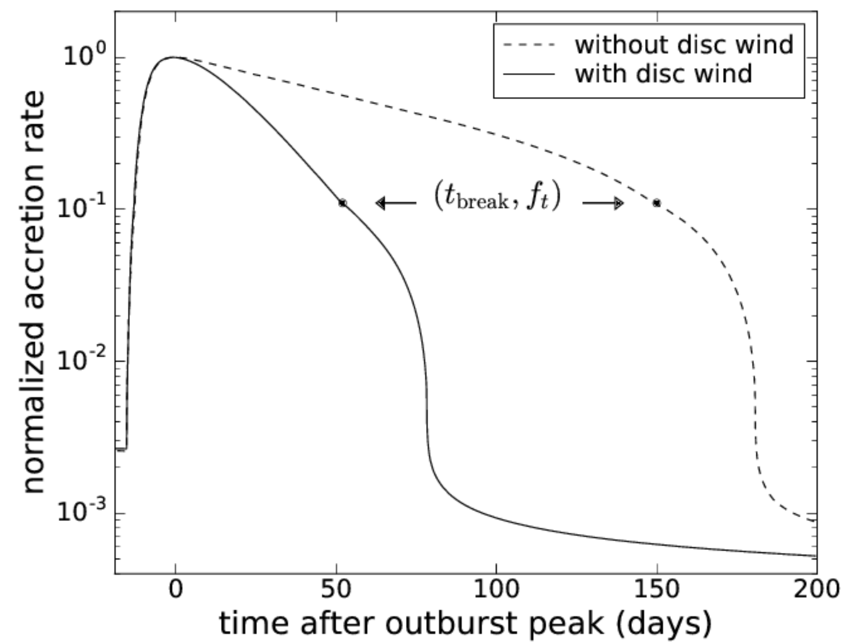


Figure 1. Overview of MAXI J1820+070 in the hard state. (a) The long term 0.2-12 keV NICER

The outbursts last longer than in CV mostly due to the irradiation effect.

On the other hand, winds can shorten the outbursts.

An example of modelling from Tetarenko et al. (2018).



Issues under discussion:
- the role of irradiation
- the role of wind
- the disk state between outbursts
- the hard/soft state transition
- the high/hard state

So what is this ?

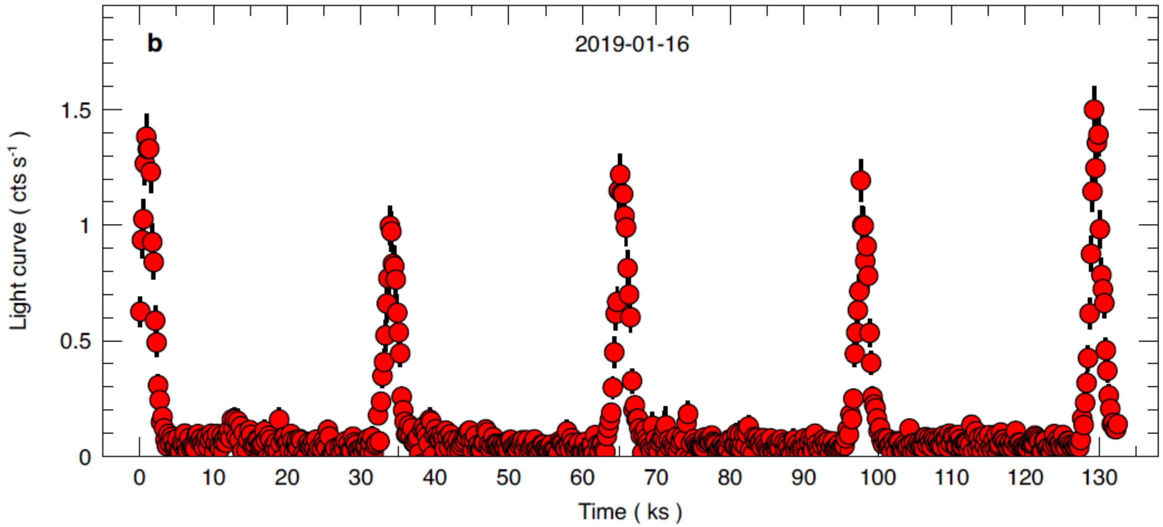


Figure 1 | X-ray QPEs in XMM-Newton and Chandra observations, 2018 December onwards

(*Miniutti et al. 2019*).

black hole mass 4 x 10^5 M_s

Fixing the radius ar 10 R_{Schw} and the mass as above we can estimate the timescales:

$$t_{dyn} = \frac{1}{\Omega_K} = 180\,s$$

$$t_{th} = \frac{1}{\alpha} t_{dyn} = 1.8\,ks$$

For α = 0.1

$$t_{visc} = \frac{1}{\alpha}\left(\frac{r}{H}\right)^2 t_{dyn} = 18\,000\,ks$$

For α = 0.1
and H/r = 0.01

It may be radiation pressure instability if we are able to shorten the viscous timescale.

?

Homework

- Show that $t_z = t_{dyn,}$ that is these two timescales are identical in the Newtonian mechanics
- Find the definition of the epicyclic frequency, explain which motion it describes