A new way to measure the neutron star parameters from atmospheric oscillations

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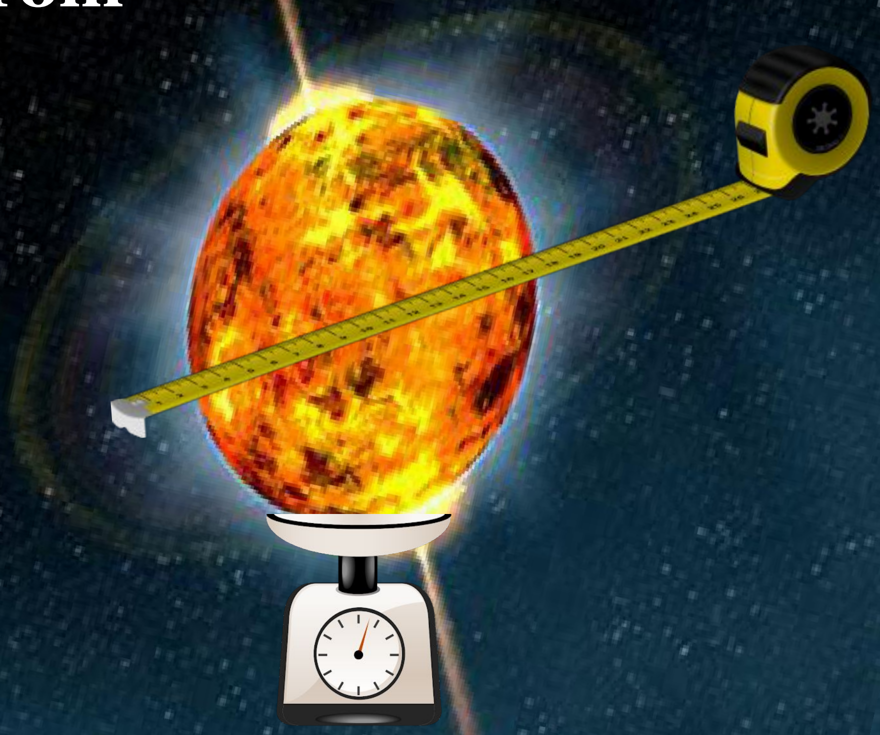
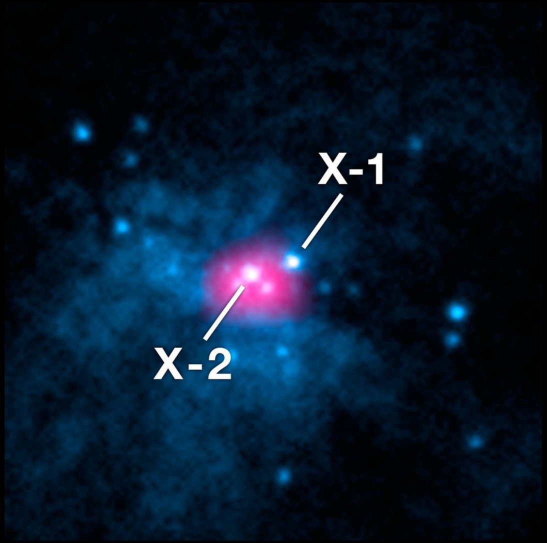


Slim accretion disks workshop.
22 October, 2018.

Neutron stars

- ➢ Neutron stars are the compact objects with supranuclear densities at the core.
- ➢ Serve as astrophysical laboratories to study the equation of state (EoS) of such dense material.
- ➢ Mass and radius are required to constrain the EoS.

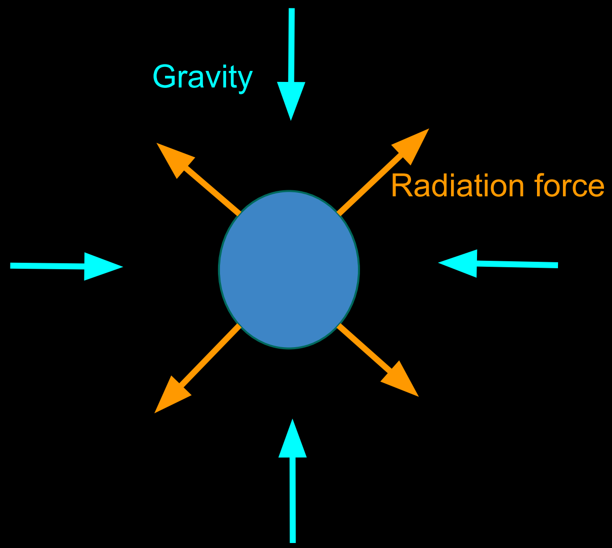


- ➢ Neutron stars are observed to be very bright, reaching near-Eddington luminosities. Two best examples:

  Type-I X-ray bursts: During the peak of the outburst.

  Ultra-Luminous X-ray sources: NGC 7793 P13, NGC 5907, M82 X-2 (NuSTAR J09551+6940.8), NGC 300 ULX1 (Bachetti et al. 2014; Israel et al. 2016, 2017)

Consider a star emitting radiation isotropically at Super-Eddington luminosity

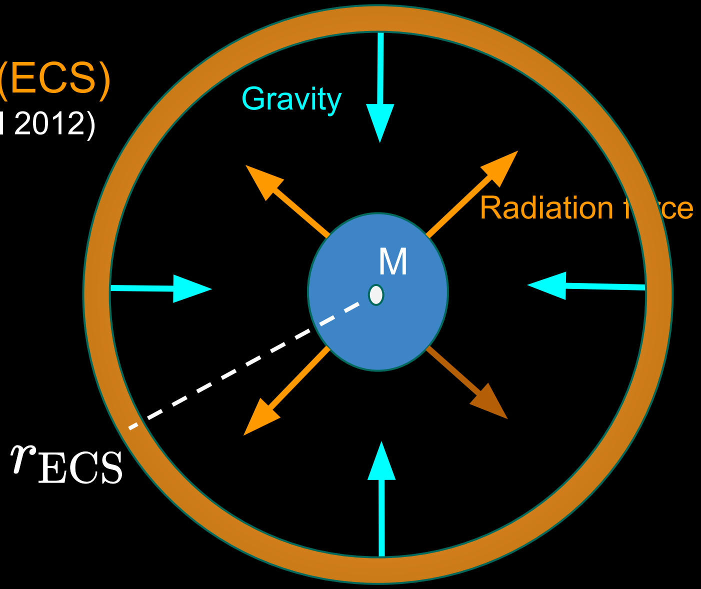


In Newtonian Theory, gravity and radiation force fall of as $1/r^2$, whereas in Theory of General Relativity, both have different radial dependence.

Consider a star emitting radiation at Super-Eddington luminosity

Levitating atmosphere
or
Eddington Capture Sphere (ECS)
(Abramowicz et. al 1990, Stahl et. al 2012)

$$r_{\mathrm{ECS}} = \frac{2M}{1-\lambda^2}$$



$$\lambda = \frac{L_\infty}{L_{\mathrm{Edd}}}$$

is the Eddington parameter

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Levitating atmospheres - Hydrostatic equilibrium

- Assume a static, spherically symmetric spacetime

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Mass conservation

$$\nabla_\mu \left(\rho u^\mu\right) = 0$$

- Energy-momentum conservation

$$\nabla_\mu T^{\mu\nu} = G^\nu \qquad \textit{for gas}$$

$$\nabla_\mu R^{\mu\nu} = -G^\nu \qquad \textit{for radiation}$$

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- Hydrostatic equilibrium

$$\boxed{\frac{1}{\rho}\frac{dp}{dr} = -\frac{M}{r^2}\left(1 - \frac{2M}{r}\right)^{-3/2}\left(\sqrt{1 - \frac{2M}{r}} - \lambda\right)}$$

Optically thin limit

Polytropic atmospheres

- The hydrostatic equilibrium condition can be analytically solved for polytropic atmospheres.

$$p \propto \rho^{\Gamma}$$

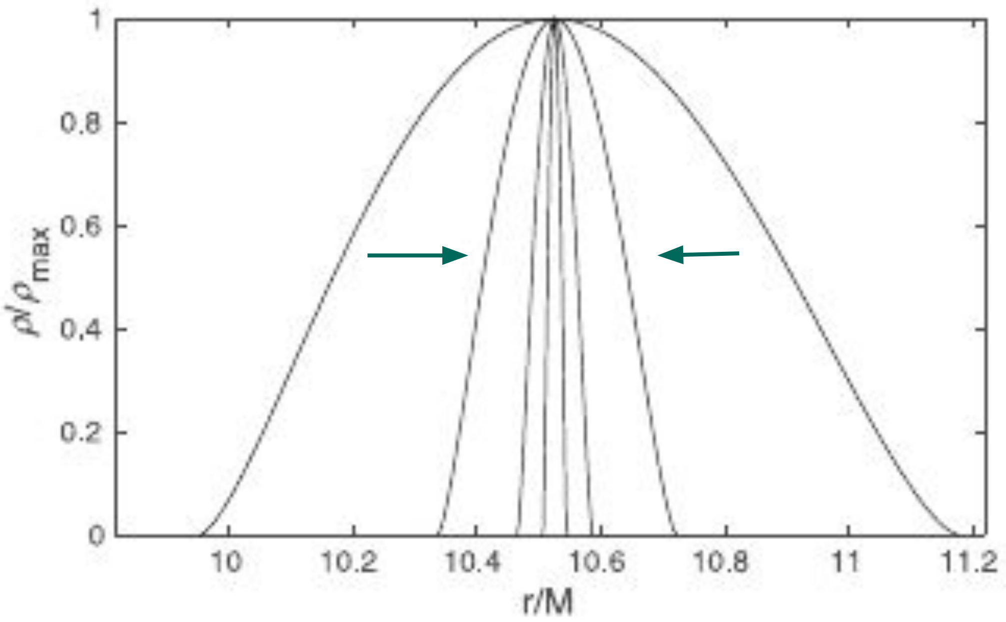


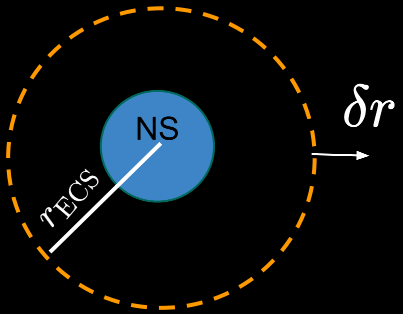
Figure 3. Density profiles of polytropic Thomson-scattering atmospheres for $\lambda = 0.9$. For a luminosity this large, the atmosphere is well separated from the neutron star surface (at $R_*/M \approx 5$). Temperatures from outside in (more extended atmospheres to less extended atmospheres) are $T_{\rm max} = 5 \times 10^7$ K, 5×10^6 K, 5×10^5 K, 5×10^4 K, respectively, with $\mu = 1/2$ and $\Gamma = 5/3$ in all cases. The density maxima are at $R_{\rm ECS} = 10.5\, GM/c^2$.

(Wielgus et. al 2015)

Radial perturbations of the levitating atmosphere

Assumption:

Geometrically thin atmosphere



Fluid variables :

$$\delta X = X(r)e^{i\omega t}$$

Radiation Drag

$$\delta \mathcal{D} = \chi \delta u^r$$

A function of radiative force terms. Under geometrically thin limit, $r \rightarrow r_{\rm ECS}$, χ is constant

Linearized conservation equations, along with the equation of state gives

$$(1-\eta^2)\frac{d^2W}{d\eta^2} - 2n\eta\frac{dW}{d\eta} + \frac{2n\omega^2\lambda^4}{\omega_r^2 g_{tt}^2}\left(1 + i\frac{\chi\sqrt{-g_{tt}}}{\omega}\right)W = 0$$

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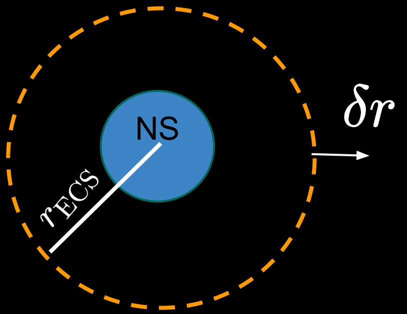

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$$-\frac{\delta p}{\rho u^t}$$

scaled radial coordinate

adiabatic index

M/r^2_{ECS} , fundamental angular frequency

- Imaginary part yields

$$i2n\left(\frac{2\omega_{\mathrm{R}}\omega_{\mathrm{I}}+\omega_{\mathrm{R}}\chi\sqrt{B})}{\omega_r^2}\right)W=0.$$

which gives the damping coefficient,

$$\boxed{\omega_{\mathrm{I}}=-\frac{\chi}{2}\sqrt{1-\frac{2M}{r}}}$$

- Real part of the eigenvalue problem is a Gegenbauer differential equation

$$(1-\eta^2)\frac{d^2W}{d\eta^2}-2n\eta\frac{dW}{d\eta}+2n\left[\frac{\omega_{\mathrm{R}}^2-\omega_{\mathrm{I}}(\omega_{\mathrm{I}}+\chi\sqrt{B})}{\omega_r^2}\right]W=0$$

and the Gegenbauer relation gives,

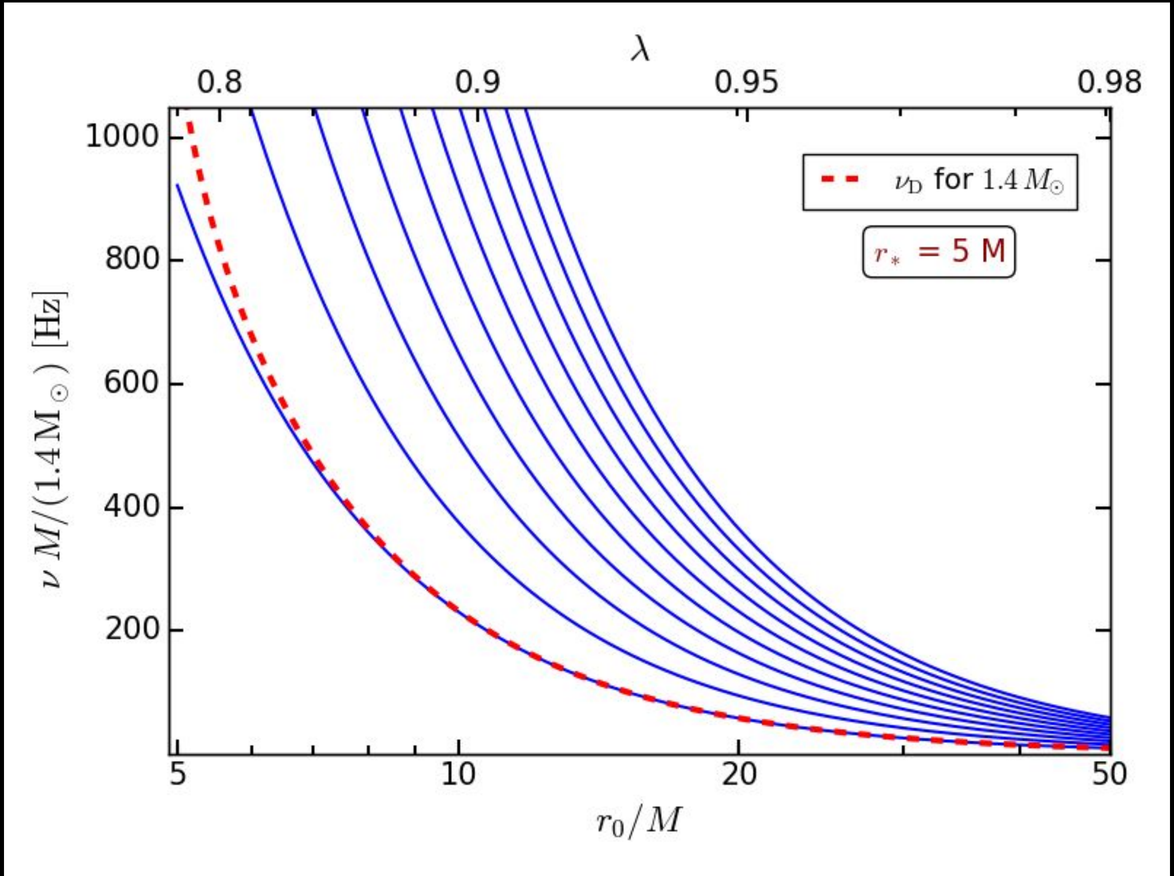
$$\boxed{\omega_{\mathrm{R}}^2=\omega_r^2\frac{k(k+2n-1)}{2n}-\frac{\chi^2}{4}\left(1-\frac{2M}{r}\right)}$$

Undamped Oscillations

$$\chi = 0 \rightarrow \omega_{\mathrm{I}} = 0$$

➢ Undamped frequencies of the ten first normal modes of the geometrically and optically thin atmospheres as a function of the atmosphere location

Frequencies of the oscillations decrease with increasing Luminosity.



Undamped Oscillations

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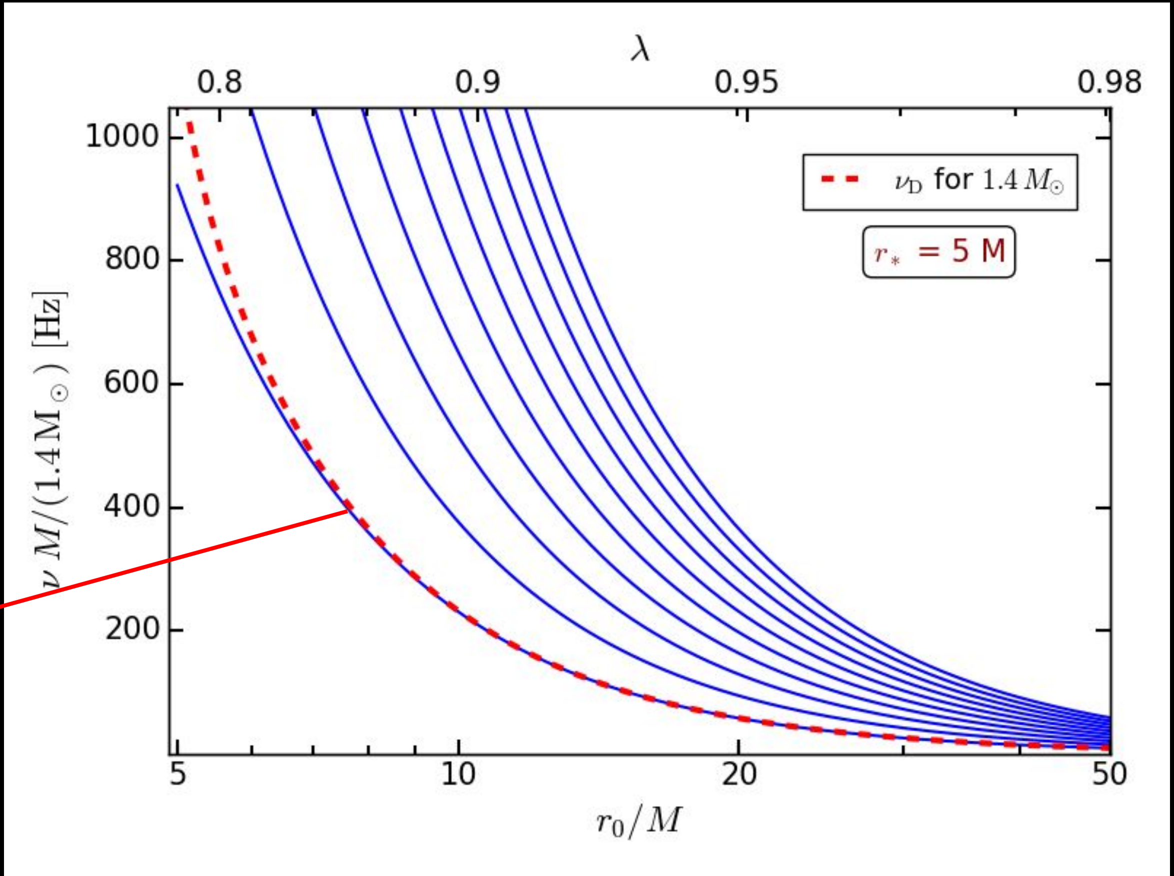
- Undamped frequencies of the ten first normal modes of the geometrically and optically thin atmospheres as a function of the atmosphere location

Damping rate

$$\boxed{\nu_{\mathrm{D}} = c^3\, \omega_{\mathrm{I}} / (2\pi\, G\, M)}$$



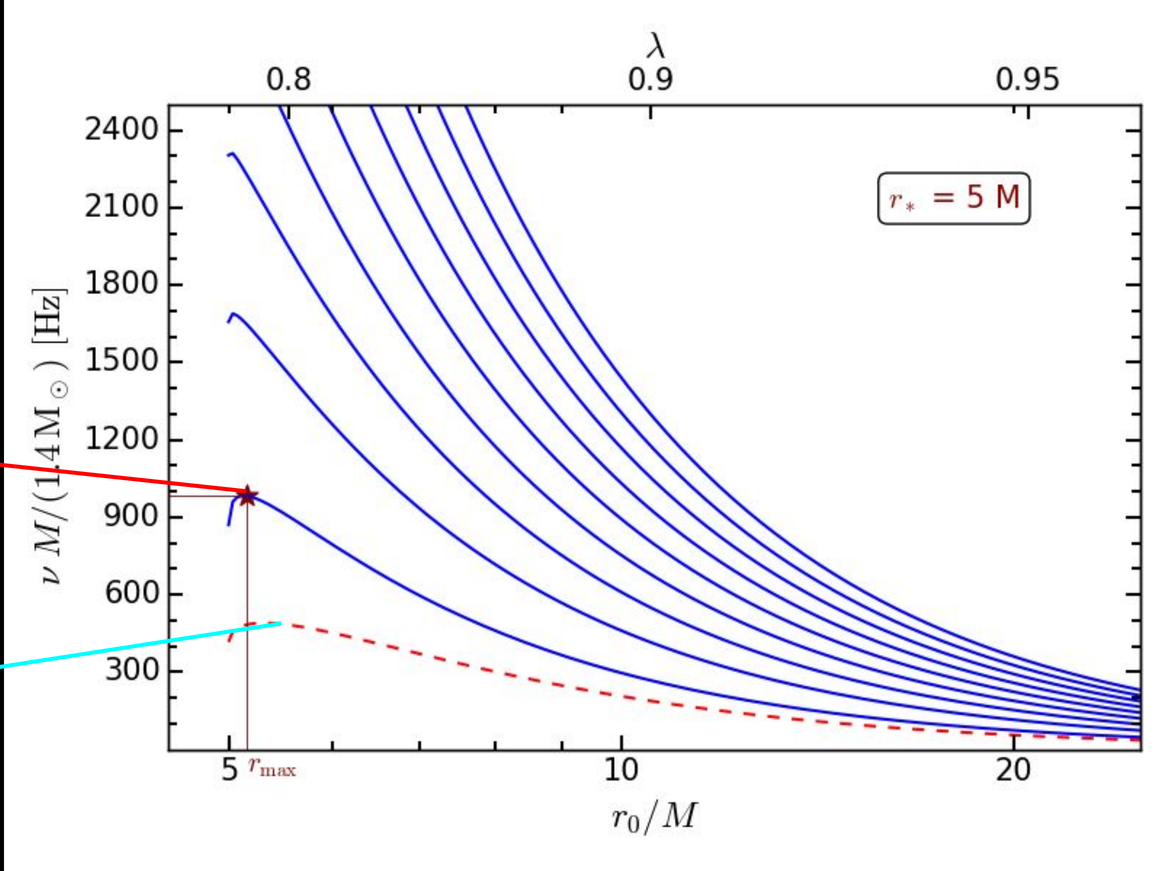
Damped Oscillations

- Damped frequencies of the first few normal modes

A **maximum** in frequency

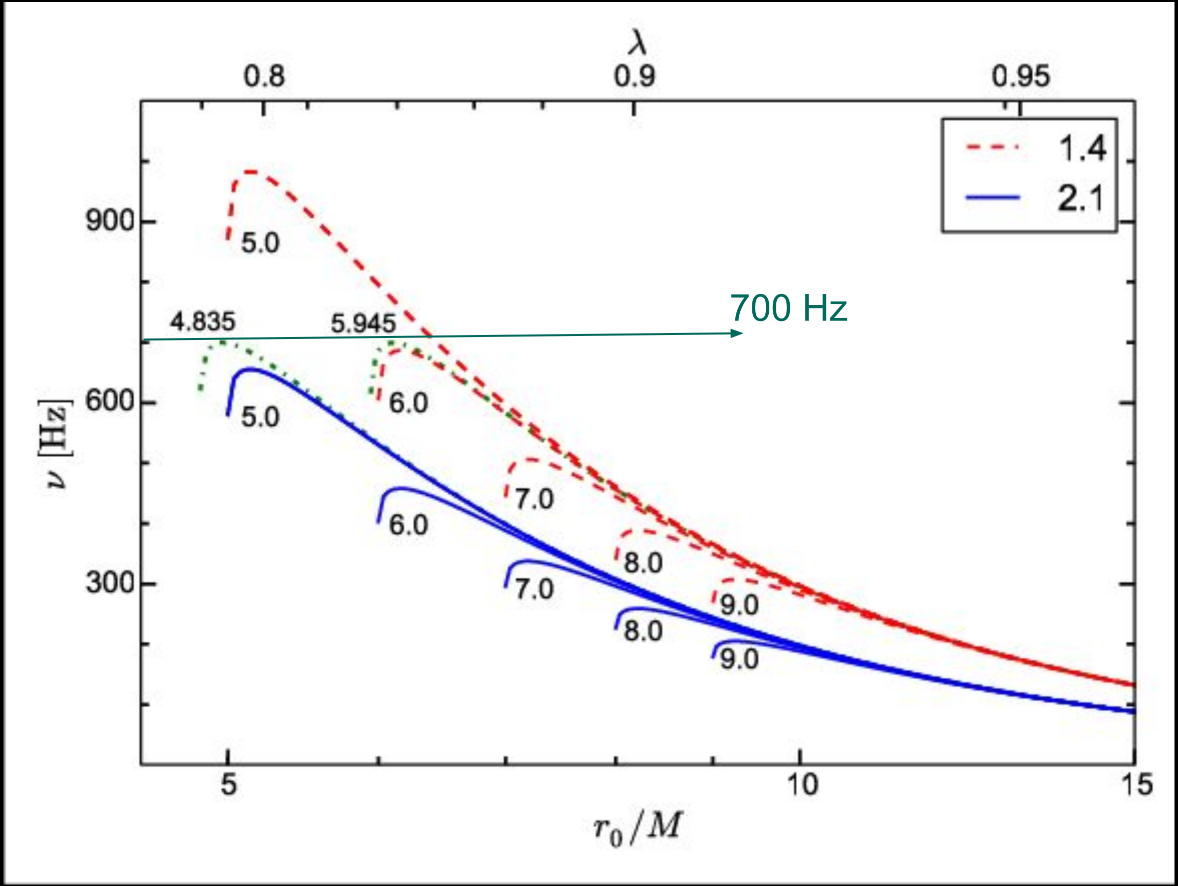
Overdamped mode

$$|\omega_{\mathrm{I}}|/2\pi$$



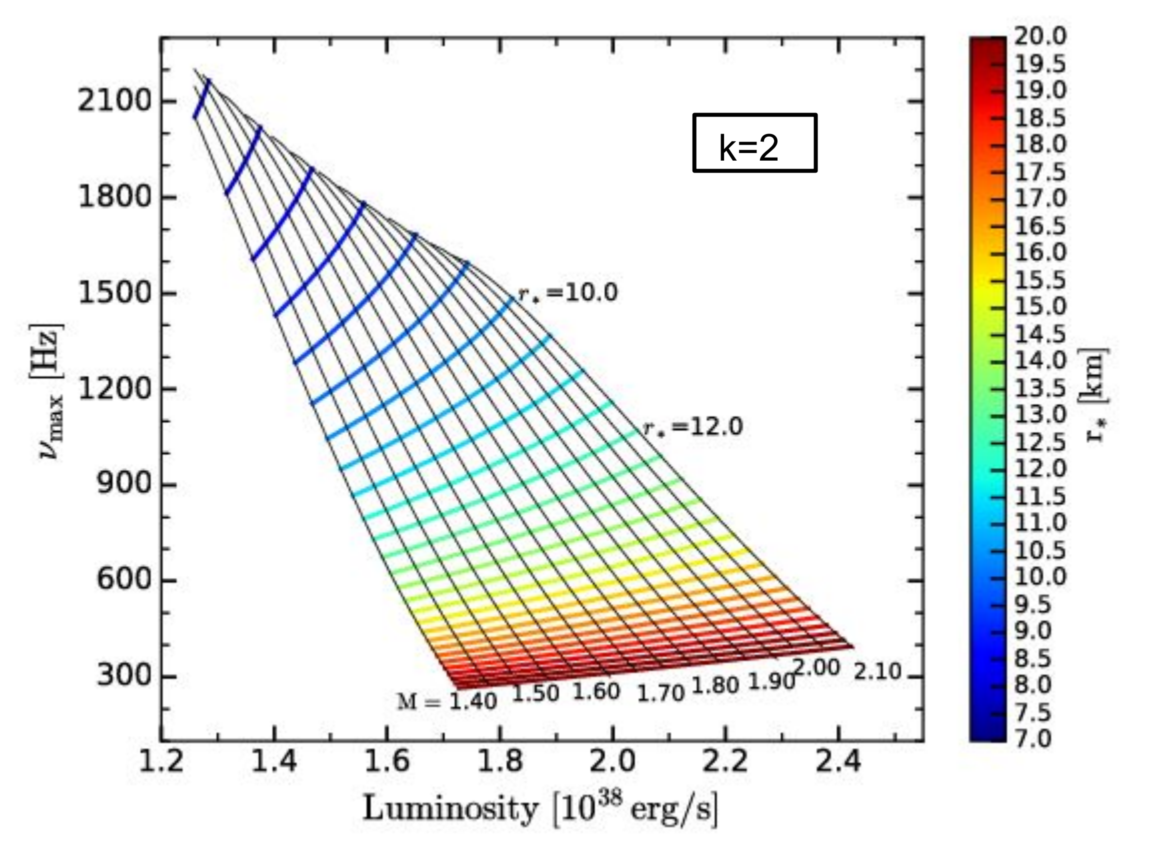
Variation of frequency maximum with stellar parameters

- Maximum is always located close to r_*, irrespective of the values of r_*/M and *M*.
- For a given *M*, $f_{\max}$ is inversely proportional to . r_*/M
- For a given r_*/M, $f_{\max}$ is again inversely proportional to *M*.
- Degeneracy in the $f_{\max}$.
- For a given r_*/M, $f_{\max}$ occurs at the same radius irrespective of *M*.



Mass and radius from the Frequency maximum

All we need are the luminosity and the frequency !



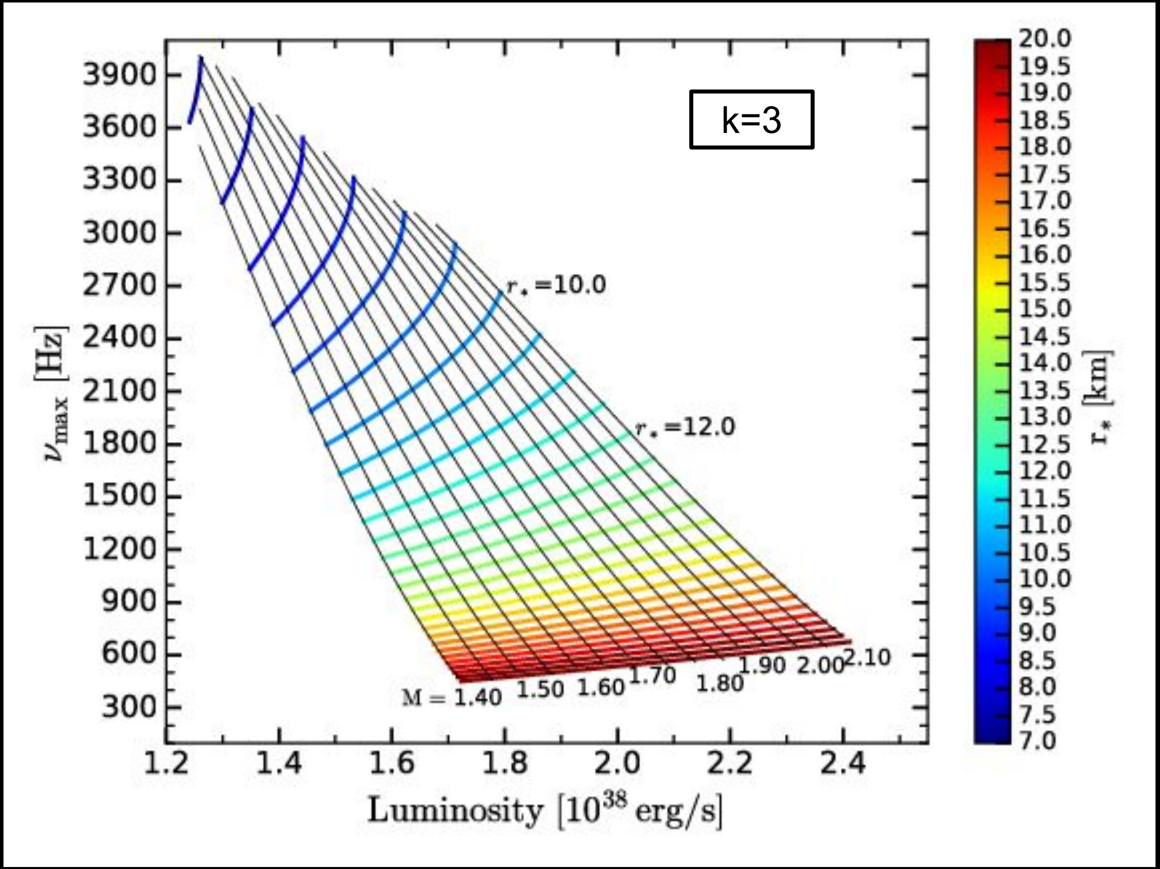
600 Hz frequency observed at a luminosity $1.9 \times 10^{38}\ \mathrm{erg\,s^{-1}}$

$$M \sim 1.65 M_{\odot}, r_* \sim 14.5\ \mathrm{km}$$

Mass and radius from the Frequency maximum

600 Hz frequency observed at a luminosity 1.9×10^{38} erg s^{-1}

$$M \sim 1.6 M_\odot, r_* \sim 18.5\ \mathrm{km}$$



Summary

- Neutron stars emitting radiation at near-Eddington luminosity harbour Levitating atmospheres.
- We investigated the radial oscillations of such atmospheres, accounting for the radiation drag.
- The frequencies of the underdamped oscillations exhibit a characteristic maximum, which is a function of stellar mass and radius.
- Based on this maximum value of frequency, and the corresponding luminosity, we derive the mass and radius of the neutron star.