Examination of Limit-Cycle behaviour using slim disc models

John Miller & Ewa Szuszkiewicz

(Oxford/Szczecin)

Work done in ~1995-2001

Study of non-stationary behaviour in accretion discs around black holes caused by *thermal instability*

- focussed on the inner part of an accretion disc around a *stellar mass* black hole
- matter *ionized and radiation dominated*
- starts off with stationary flow, geometrically thin and optically thick
- *but* thermally unstable

Can it produce a limit cycle?

[study following on after Honma, Matsumoto & Kato 1991]

Mechanism: a local increase in temperature:

→ expansion

→ increased radiation losses

→ increased viscous heating

(Increased cooling) > (Increased heating)

→ *stable* situation

(Increased heating) > (Increased cooling)

→ *instability*

Our strategy:

- Follow the Slim Disc approach
- Use Lagrangian finite difference hydro code with adaptive gridding (*D/Dt* denotes a Lagrangian, comoving, time derivative in the following)
- Start from the simplest picture and add improvements one by one

Starting approximations (staying close to the the original Slim Disc approach):

1) Gravity represented by the Paczyńsky-Wiita pseudo-Newtonian potential

2) Vertically integrated, non-self-gravitating models

3) Assume vertical hydrostatic equilibrium

4) alpha-law viscosity with $\alpha = 0.001$

5) Optically thick; $p = k\rho T + aT^4/3$

We then solve the following system of equations →

Mass conservation equation

$$\frac{D\Sigma}{Dt} = -\frac{\Sigma}{r}\frac{\partial}{\partial r}(r v_r)$$

Radial equation of motion

$$\frac{D v_r}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\partial \Phi}{\partial r} + \frac{l^2}{r^3}$$

Azimuthal equation of motion

$$\frac{Dl}{Dt} = \frac{\alpha}{r\Sigma}\frac{\partial}{\partial r}(r^2 P)$$

Vertical equation of motion (*gives the disc height* *H*)

$$\frac{D v_z}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\partial \Phi}{\partial z} \quad \rightarrow \quad 0$$

Energy equation (solved implicitly for *T*)

$$\rho T\frac{DS}{Dt} = Q_{vis} + Q_{rad}$$

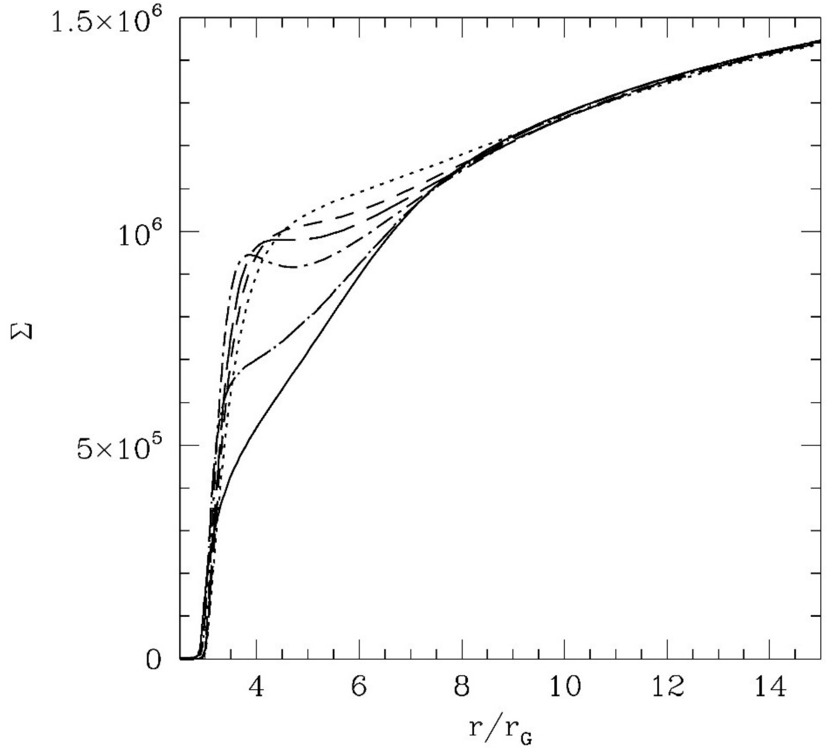
Initial data:

- Calculate initial stationary transonic model using Ewa's (1988) code
- Parameters chosen so that this had a thermally unstable region (according to local stability analysis)
- Transfer data to the finite-difference grid for the time-dependent code
- Noise generated was sufficient to produce suitable generalised perturbations for triggering growth of globally unstable modes (if present)

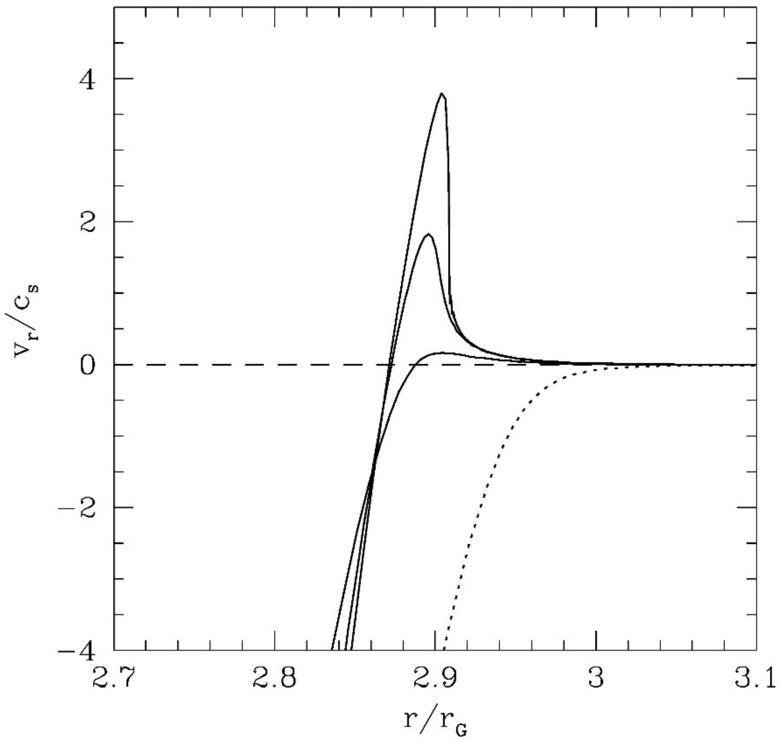
Results for the first set of models

- These all had $\alpha = 0.001$ and $M_{BH} = 10M_{\odot}$
  - for $L < 0.07\ L_E$ they are thermally stable
- Focus on a representative unstable model with $L = 0.1\ L_E$ (unstable region from $4.6 – 17.5\ r_G$ according to local linear analysis)
- No limit cycle behaviour seen despite initial growth of possibly promising behaviour
- Progress stopped by a violent instability triggered when the disturbance from the original unstable region reached the sonic point (at $r < 3r_G$)

α = 0.001, M_{BH} = 10$M_{\odot}$, L = 0.1 L_E



0 < t < ~180s



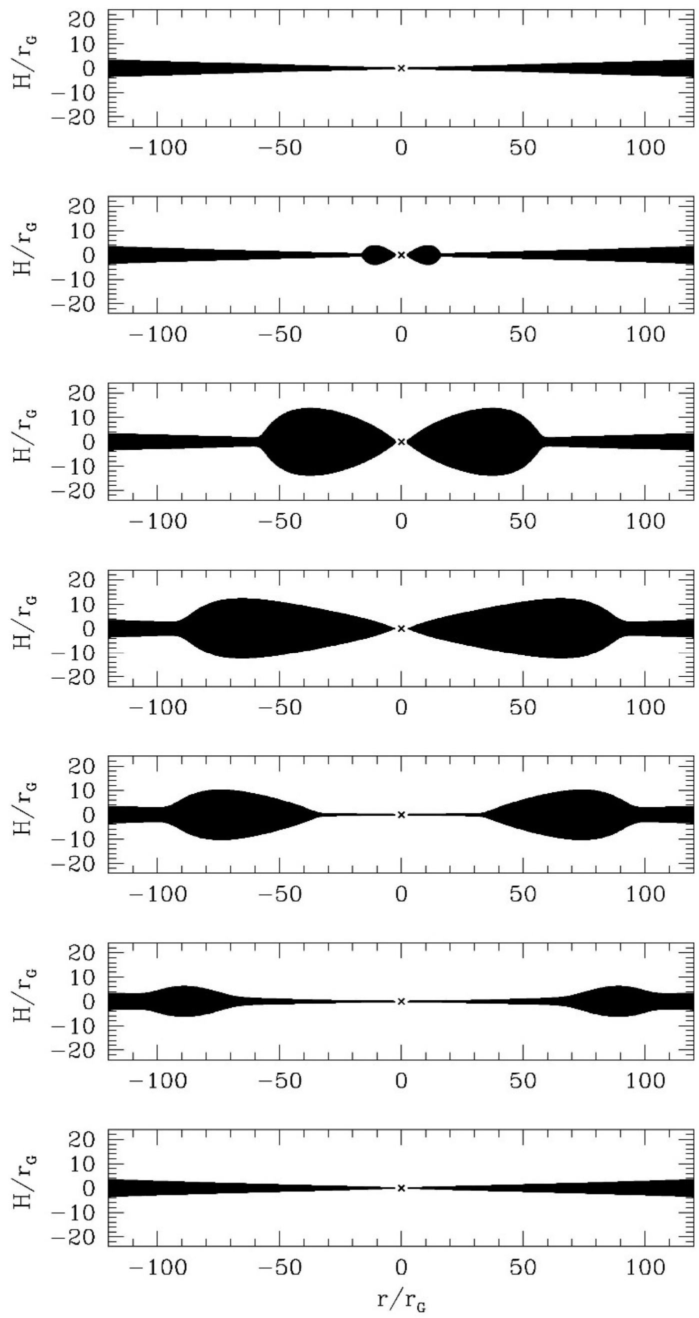
Next $\Delta t \sim 7 \times 10^{-3}$s

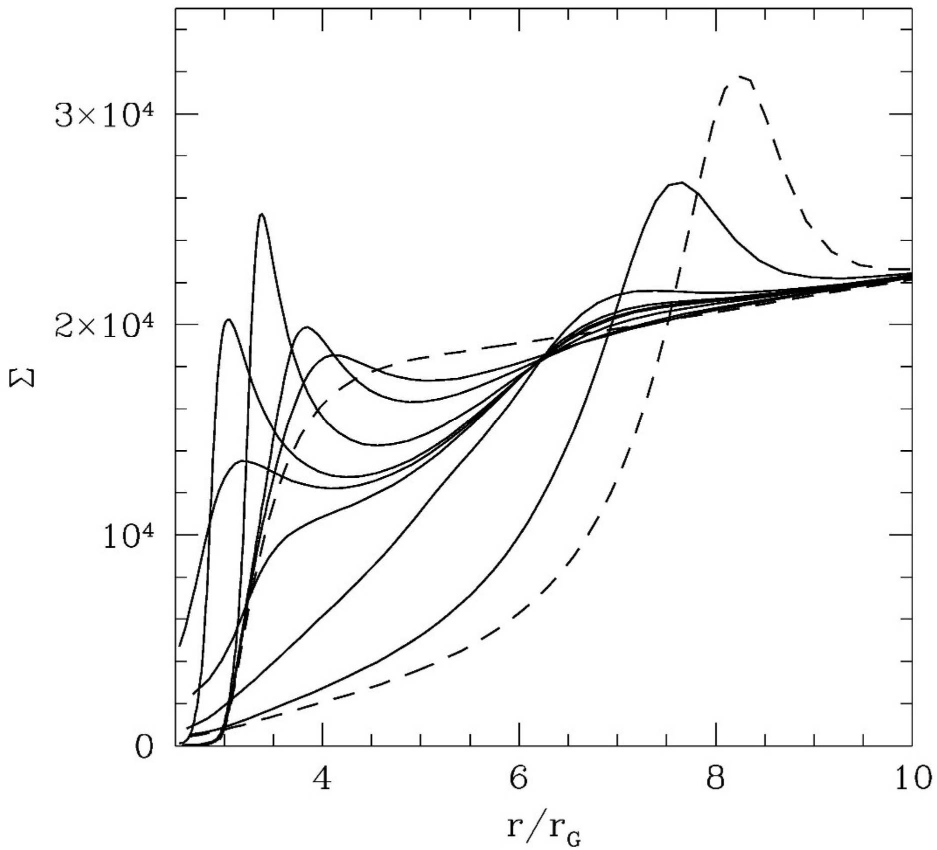
Second step:

- We then increased α to 0.1, but kept other basic things the same
- Focussed on an initial model similar to the one of Honma et al for which they did find limit cycles:

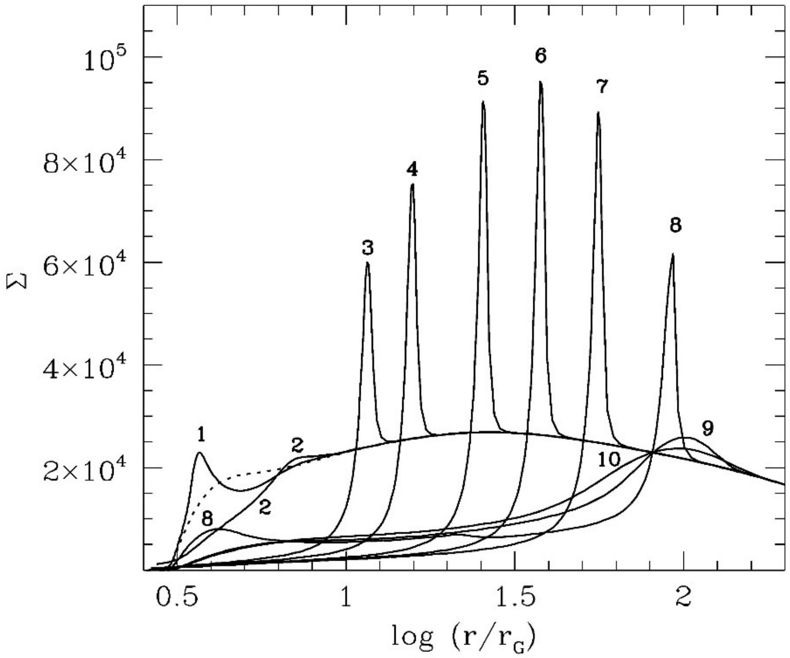
$$\alpha = 0.1 \qquad \frac{\dot{M}}{M_{cr}} = 0.06 \quad \left(= \frac{L}{L_E}\right)$$

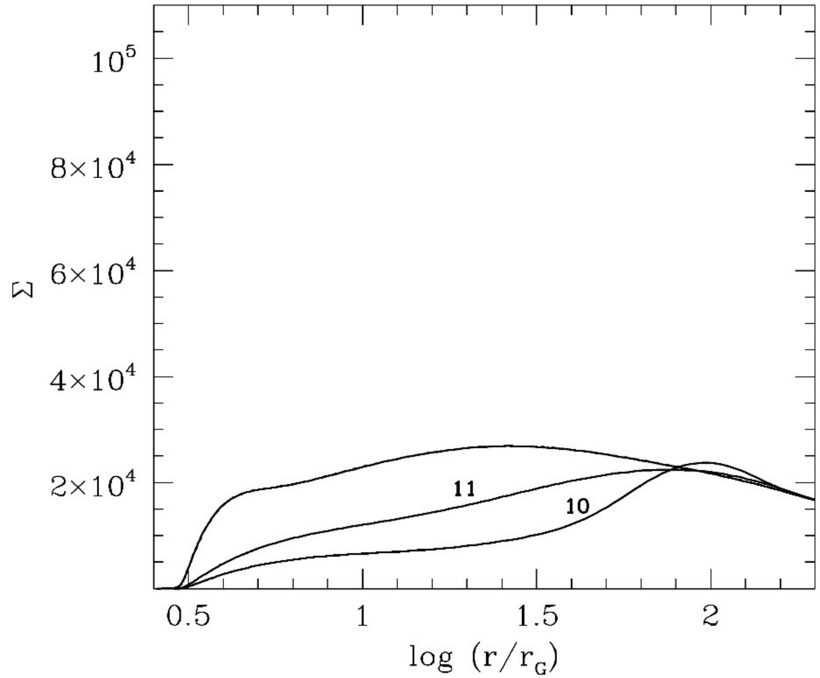
- The unstable region for this is from *4.5 – 17.5* r_G , almost exactly the same as before


H/r_G
r/r_G
−100
−50
0
50
100
20
10
0
−10
−20

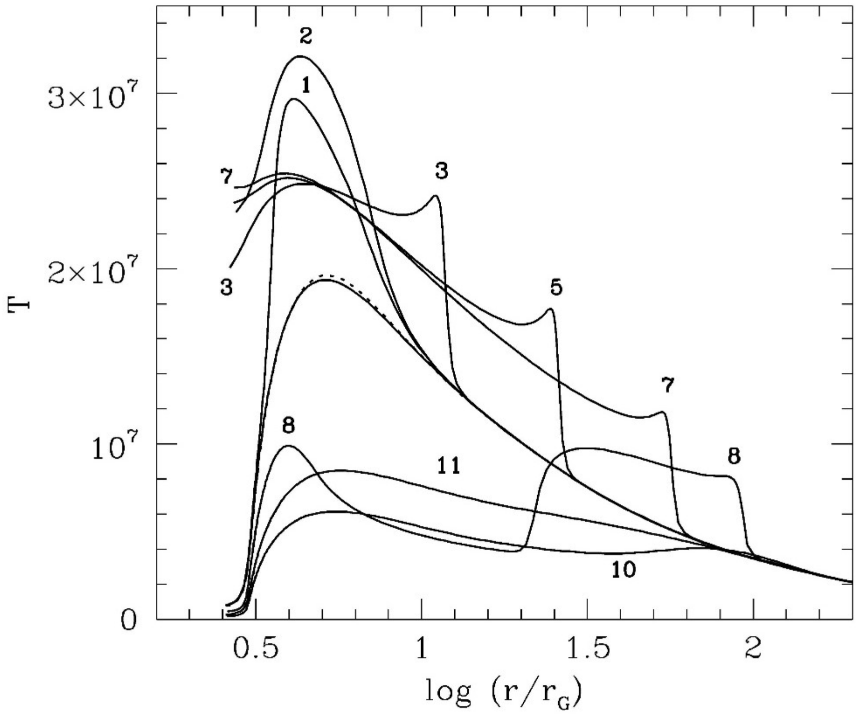


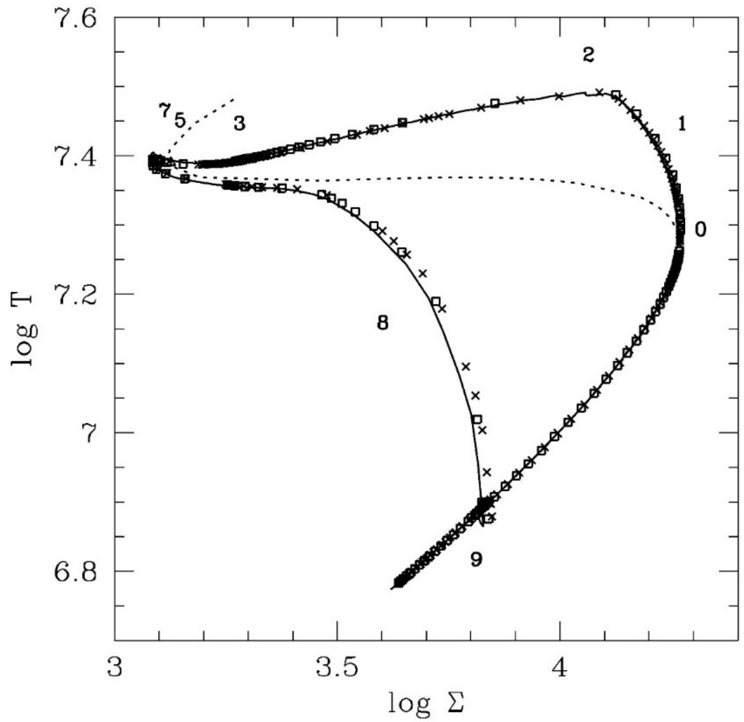
3×10^4
2×10^4
10^4
0
Σ
4
6
8
10
r/r_G





Time for one cycle: ~ 800s (Outburst: ~ 20s)



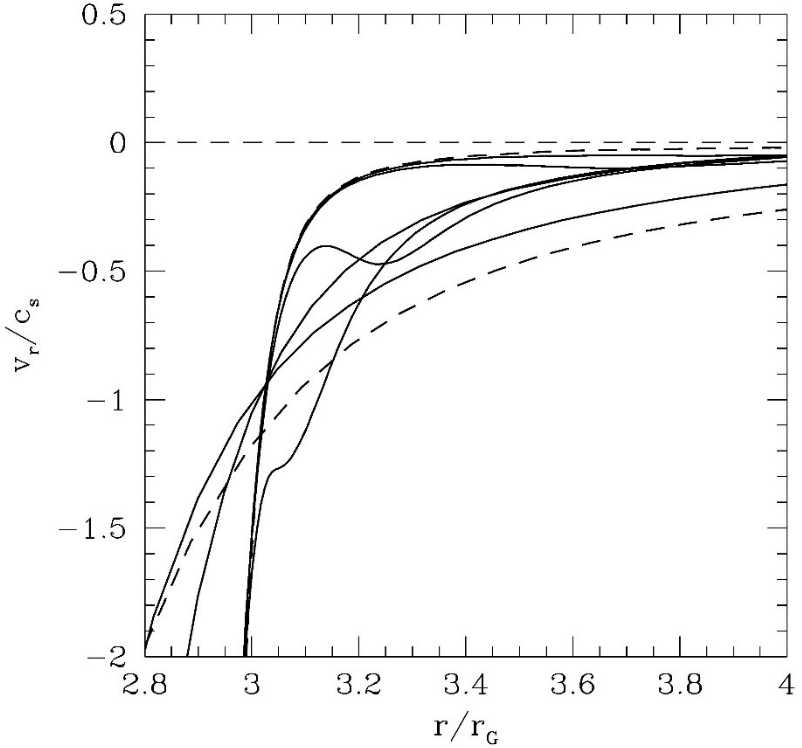


Phase portrait at $5r_G$

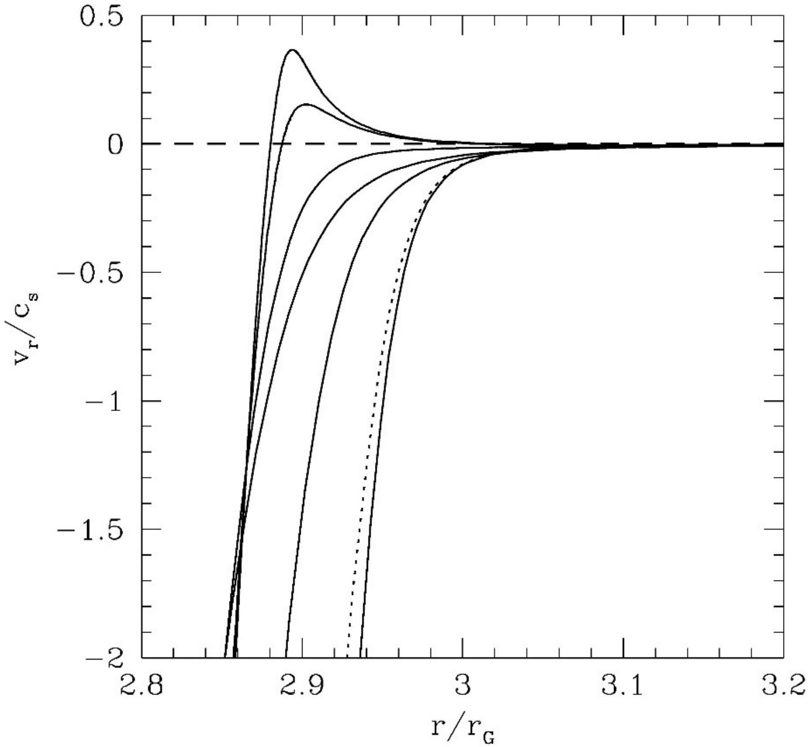
Further investigations

- Use of diffusive viscosity rather than an alpha law
- Using a dynamical equation for the vertical acceleration
- Effects of departures from optical thickness
- Continuing for many cycles and showing that the behaviour was not just deviations centred on the stationary solution

This was all interesting but did not fundamentally change the overall picture given above



$\alpha = 0.1$



$\alpha = 0.001$

Conclusions:

- We were in substantial agreement with the results of Honma et al for the higher alpha case but were adding context and some new insights
- A key issue for whether limit cycles occurred or not concerned whether the sonic point is inside or outside $r = 3r_G$
- This work was of its time and things seem to have moved on since then