Tidal disruption and slim discs

J. C. B. Papaloizou

with P. Ivanov V. Zhuravlev
M. Xiang - Gruess

SLIM ACCRETION DISKS

M. A. Abramowicz,[1,2] B. Czerny,[1,3] J. P. Lasota,[1,4] and E. Szuszkiewicz[1]

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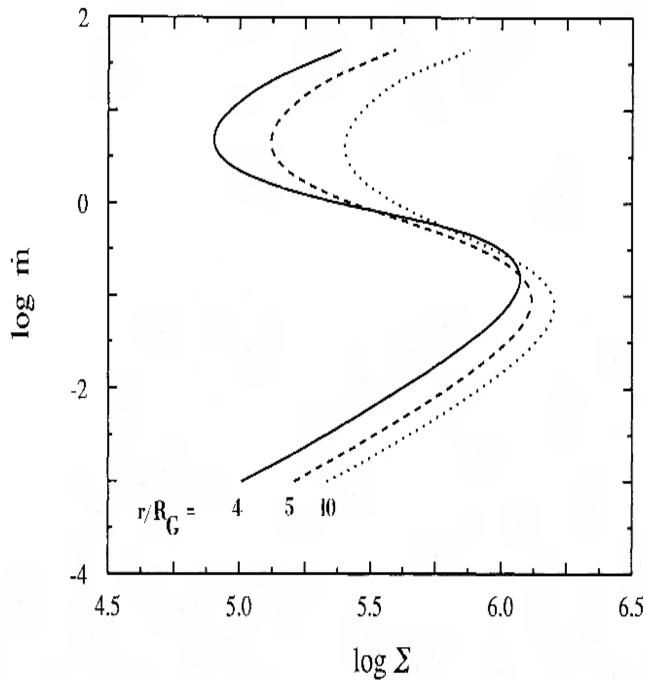
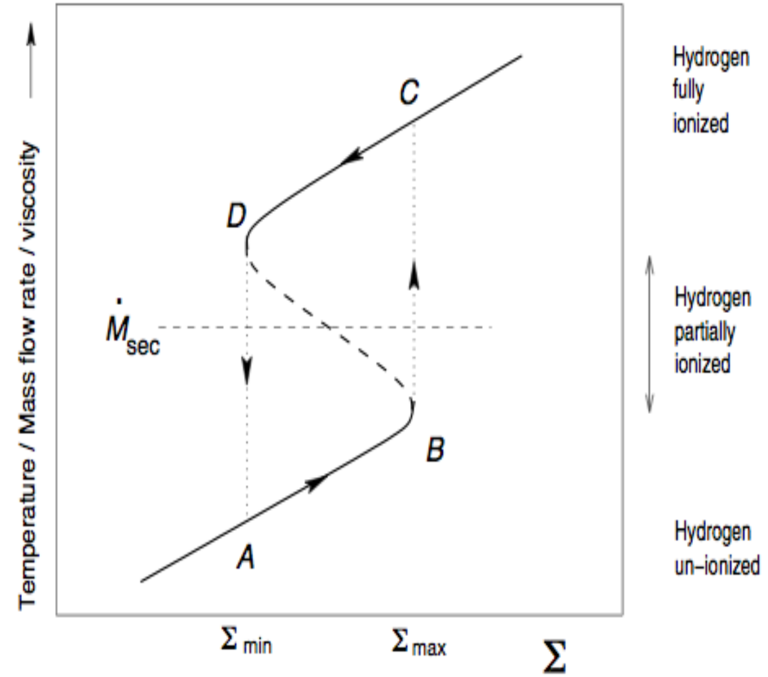


Fig. 2.—The $\dot{m}(\Sigma)$ relation for slim accretion disk models for three fixed radii, $r/R_G = 4$ (*solid line*), 5 (*dashed line*), and 10 (*dotted line*). Σ is a surface density in g cm^{-2}.

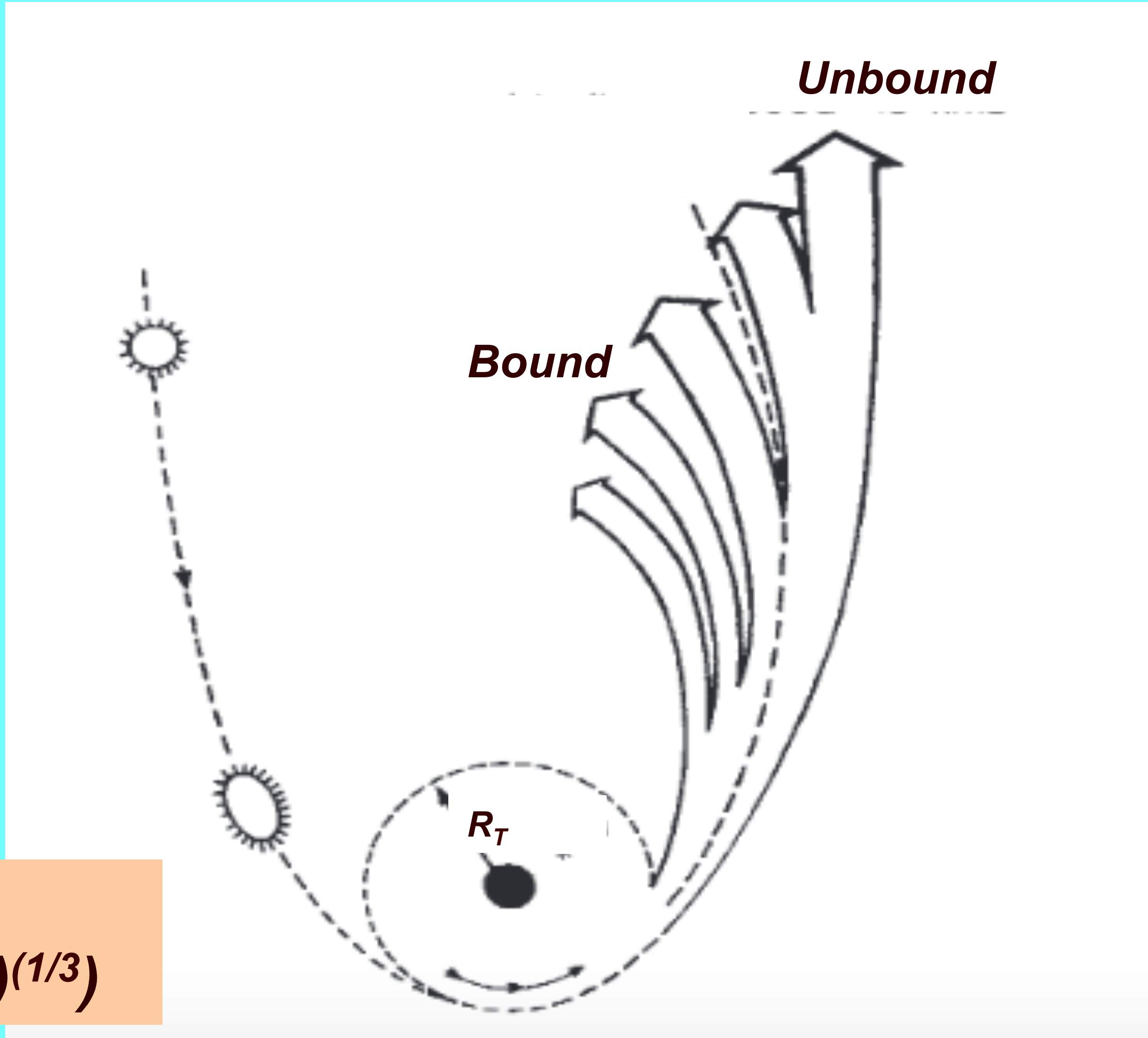


We study the evolution of disks formed by the tidal disruption of a star by a black hole.

We consider the case when the disk becomes warped on account of a misalignment between the black hole equatorial plane and the orbital plane of the stellar orbit .

We employ semi-analytic methods with SPH and grid based simulations

Tidal disruption of star on a parabolic orbit with pericentre distance equal to the tidal radius



Tidal radius
$R_T = R_* (M/m)^{(1/3)}$

Some basic length and time scales

Tidal radius $R_T = (M/m)^{1/3} R_* = 7 \cdot 10^{12} M_6^{1/3} cm$

The penetration factor for pericentre distance $B_p = R_T / R_p$

The penetration factor for stream circularization radius
$B_S = R_T / R_S$ (normally $R_S = 2R_P$)

There are also the two dynamical times

$$t_S \equiv \Omega_S^{-1} = \frac{R_S^{3/2}}{\sqrt{GM}} = \frac{R_*^{3/2}}{\sqrt{Gm}} B_S^{-3/2} \equiv B_S^{-3/2} t_* = 1.6 \cdot 10^3 B_S^{-3/2} s.$$

and

$$t_p \equiv \Omega_p^{-1} = \frac{R_p^{3/2}}{\sqrt{GM}} = \frac{R_*^{3/2}}{\sqrt{Gm}} B_p^{-3/2} \equiv B_p^{-3/2} t_* = 1.6 \cdot 10^3 B_p^{-3/2} s.$$

Accretion rate produced by stream material returning to circularization radius after closest approach

After closest approach bound stream material returns to the circularization radius, providing an accretion rate onto the black hole.

The minimum return time after the first closest approach occurs for the most strongly bound stellar material and is gven by.

$$P_{min} = \frac{\pi}{\sqrt{2}}(R_p/R_*)^3\,(m/M)^{1/2}\,t_* = 3.5\cdot 10^6 M_6^{1/2} B_p^{-3} s.$$

This corresponds to the orbital period for a semi-major axis

$\sim R_P^2/(2R_*)$.

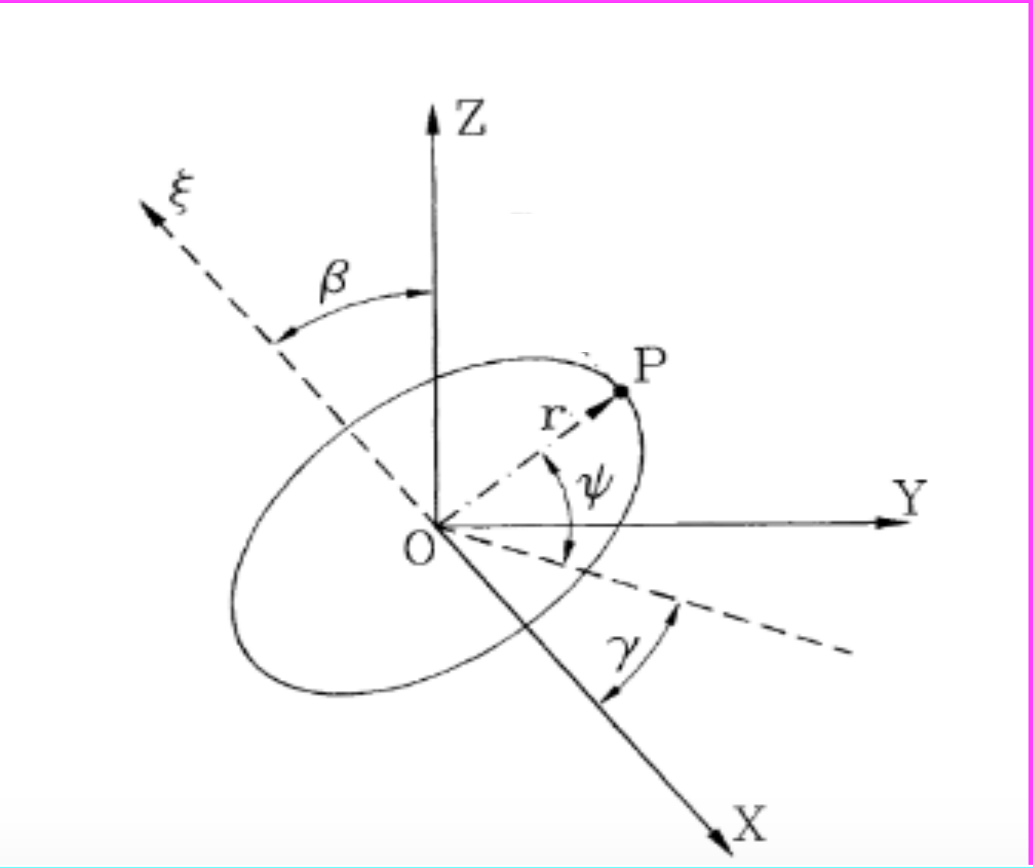
Assuming the stellar specific binding energy is a linear function of mass, the disc accretes matter from the steam at an estimated rate

$$\dot{M}_S = \frac{0.5m}{P_{min}}\left(\frac{t}{P_{min}}\right)^{-5/3} = 2.8\cdot 10^{26} B_p^3 \left(\frac{t}{P_{min}}\right)^{-5/3} g/s$$

Note that this normally exceeds the Edington limit

$$\dot{M}_{\rm E} \approx 1.7\times 10^{23} M_6\,{\rm g\,s^{-1}}$$

Geometry of disk ring inclined to black hole orbital plane



The (X,Y) plane coincides with the black hole equatorial plane
Inclination of the orbital plane of a local disk ring to the (X,Y) plane is β. Angle between X axis and line of nodes in the disk ring plane is γ.

Dynamics and governing equation for linear warps sourced by stream impact

Set $W = \beta \exp(i\gamma)$ ***and*** $W_* = \beta_* \exp(i\gamma_*)$ ***(corresponding to the stream)***

$$\dot{\mathcal{W}} = \frac{\delta^2\sqrt{GM}}{4\alpha\xi R}\frac{d}{dR}\left(\xi R^{3/2}\frac{(1+ik)}{(1+k^2)}\frac{d\mathcal{W}}{dR}\right) + i\Omega_{LT}\mathcal{W} + \dot{\mathcal{W}}_* \quad (\bigstar)$$

Where $\delta = HR$, $\xi = \Sigma\delta^2 R^{1/2}$

$$k = \frac{3GM}{c^2\alpha R}$$

and the Lens-Thirring precesssion frequency

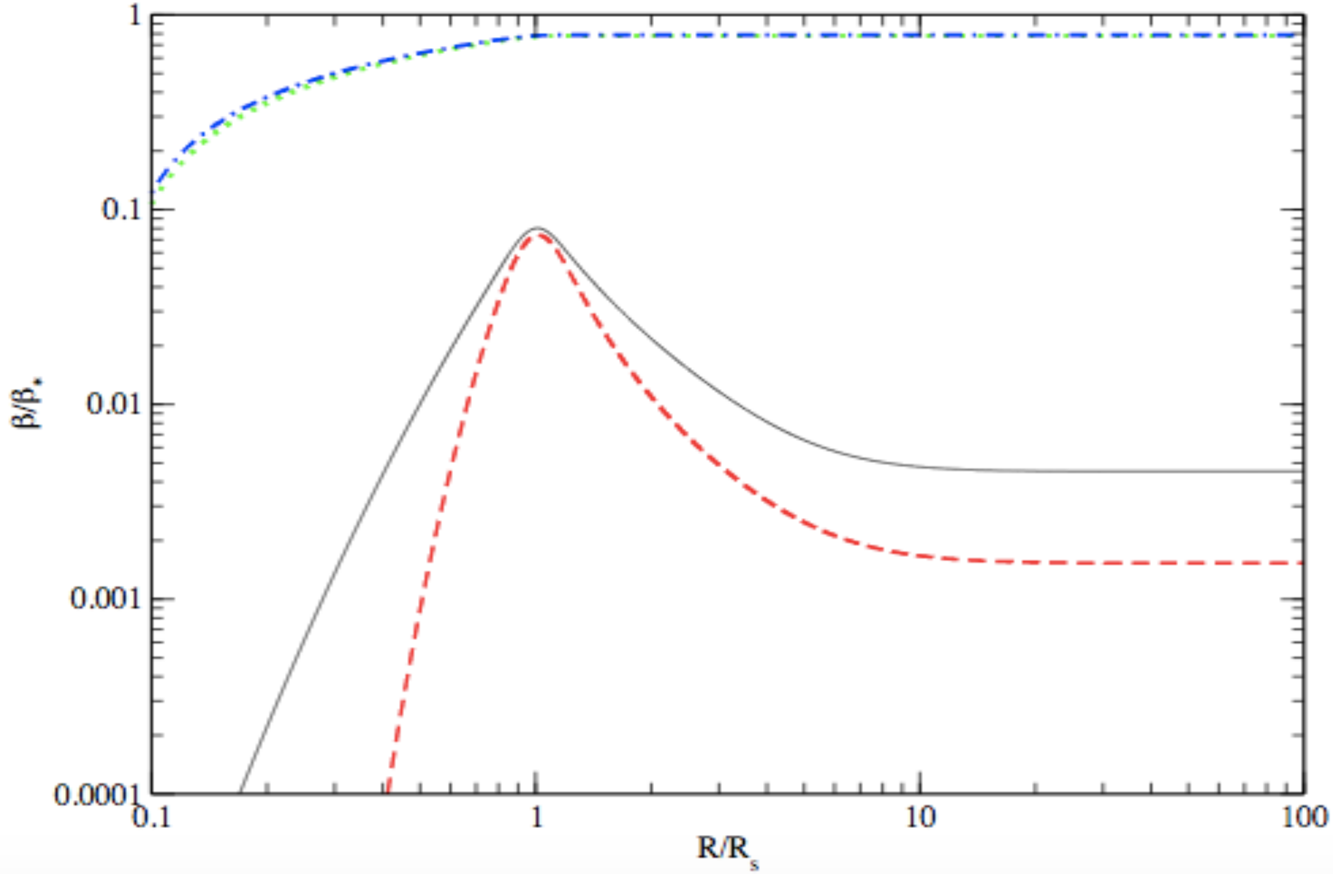
$$\Omega_{\rm LT} = 2a\frac{G^2M^2}{c^3R^3}$$

While

$$\dot{\mathcal{W}}_* = \frac{\dot{M}_{\rm S}}{2\pi\Sigma R^2}(\mathcal{W}_* - \mathcal{W})\delta_\Delta(1 - R/R_{\rm S}).$$

represents the effect of the misaligning torque due to stream

Steady state disk Inclination for semi - analytic model as a function of R /R_s



$\alpha = 0.5, \delta = 0.056, |a| = 1, M_6 = 0.02$ dotted,dot-dashed (R)

$\alpha = 0.1, \delta = 0.01, |a| = 1, M_6 = 1$ Solid,dashed (R)

Parameters for SPH simulations and grid based simulations where applicable

Black hole mass $10^6\ M_{sun}$

Disrupted star of $1M_{sun}$ - orbital inclination = $\pi/4$

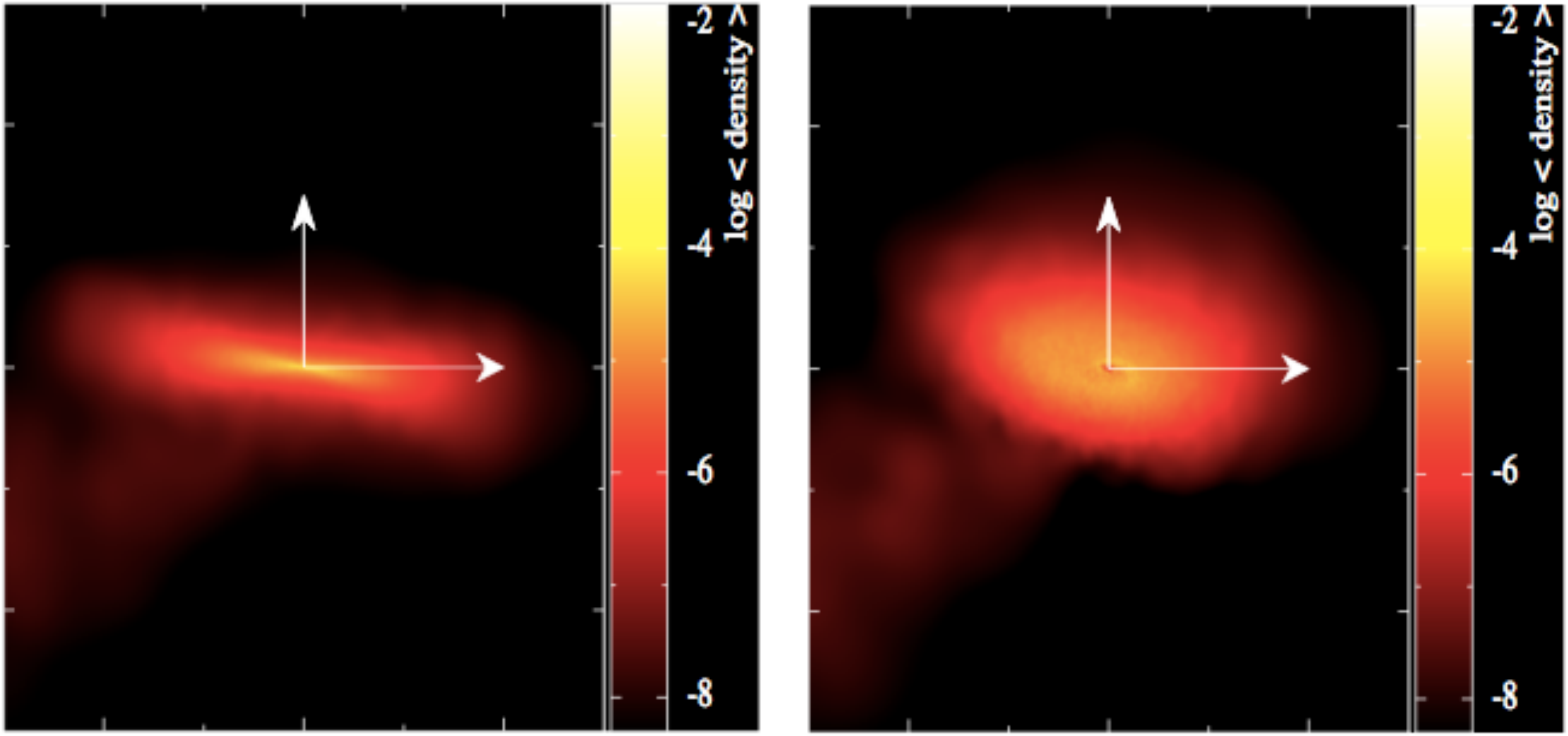
Locally isothermal equation of state for SPH

δ = H/R = 0.1, α = 0.1 from fit to free disk evolution.

Initial disc mass = $0.2M_{sun}$ for grid based work

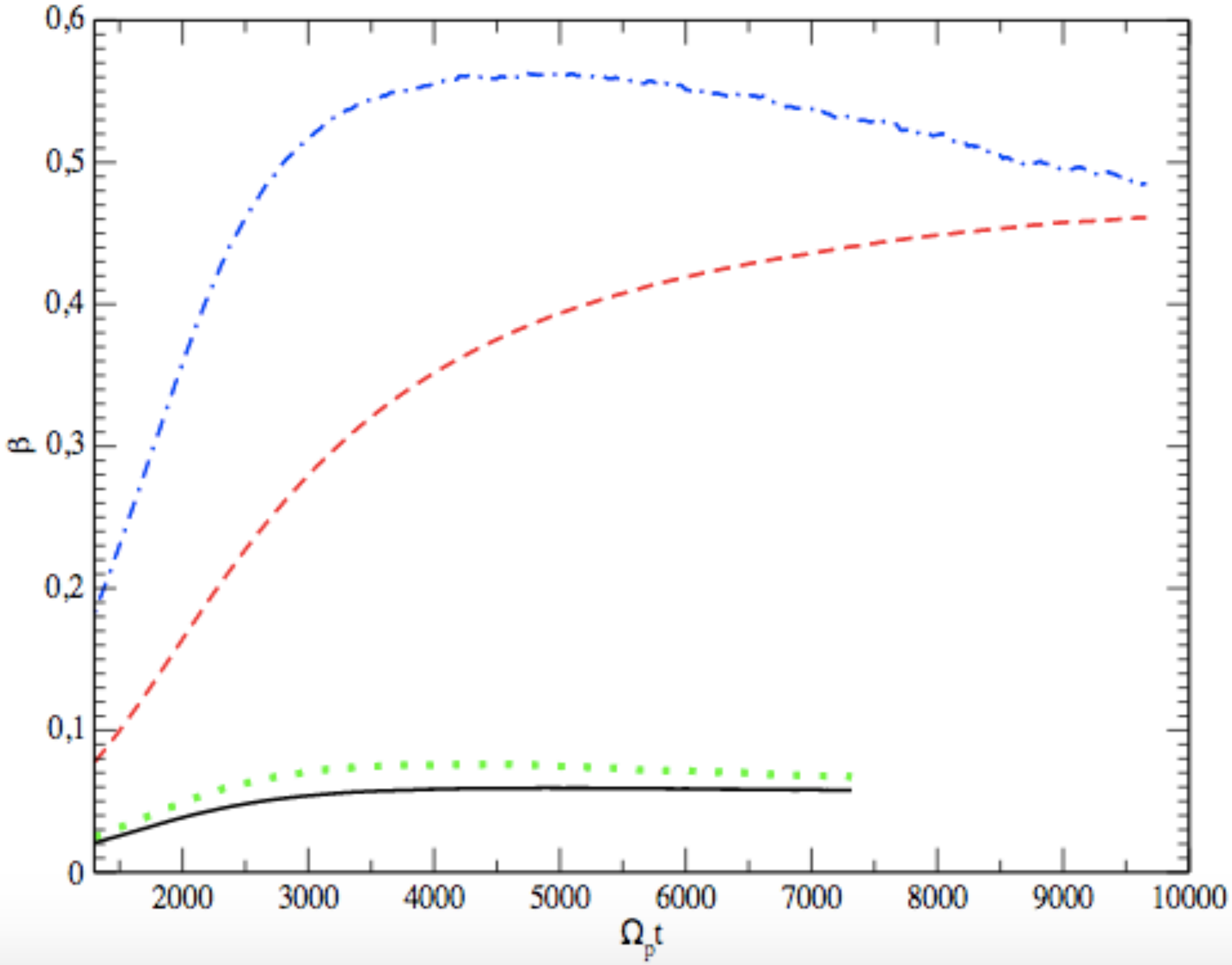
The penetration factor $B_p = R_T / R_p$ = 5/3
$R_S = 2R_p$ and energy dissipation due to circularization included for grid based work

Warped disc produced by stream misaligned with black hole equatorial plane



The disc and stream at $t=8000\ \Omega_p^{-1.}$ The vertical axis is aligned with he black hole spin. The other axis is the X axs. Tick mark separation $10R_p$
The black hole rotation parameter a =1 left panel , a =0.1 right panel .

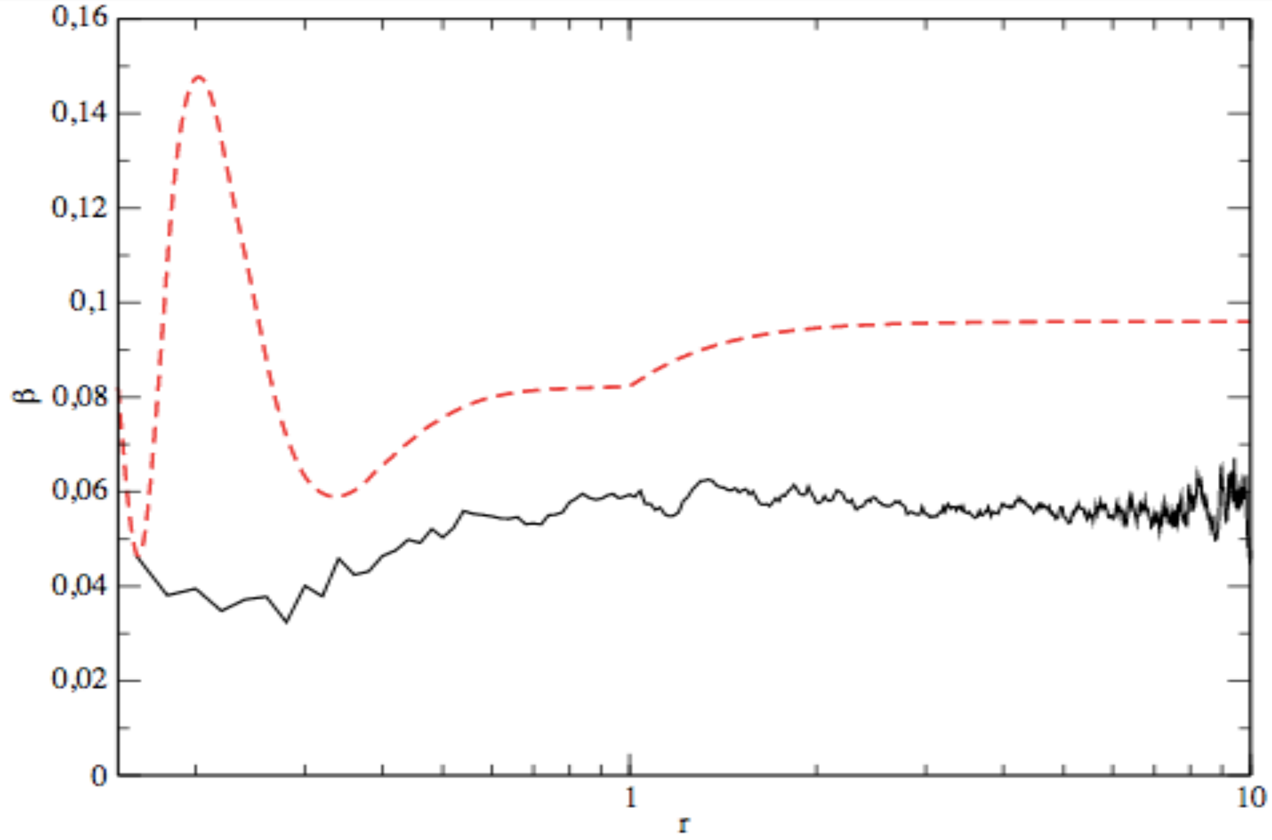
Disk inclination at stream impact radius as a function of time (SPH and semi - analytic)



SPH a=1 solid, a =0.1 dashed

Semi-analytic -- dotted,dot-dashed

Disk inclination as a function of radius after several P_{min} (SPH and semi - analytic)



The inclination angle β, in radians, shown as a function of radial distance r_p for the case with $a = 1$ at time $t = 3500\Omega_p^{-1}$.

The solid curve is from an SPH simulation, while the dashed one is obtained by solution of equation (★),

SPH evolved further towards alignment

Basic equations for vertically integrated 1D model disc − Radiation, advection and viscosity included

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \boldsymbol{v}) = S_m \, ,$$

$$\Sigma \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = -\nabla \Pi - \Sigma \nabla \Phi + \boldsymbol{f}_v ,$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E \boldsymbol{v}) = -\Pi \nabla \cdot (\boldsymbol{v}) + \epsilon_v - 2F_+ - \nabla \cdot (2HF\hat{R}) + S_{\mathrm{E}}.$$

Here, $\boldsymbol{v} = V_{\mathrm{R}} \hat{\boldsymbol{R}}$ is the velocity, with $\hat{\boldsymbol{R}}$ being the unit vector in the radial direction, Π is the vertically integrated pressure and E is the vertically integrated internal energy per unit volume. The viscous force per unit area is $\boldsymbol{f}_v$, the rate of energy input per unit area due to viscous dissipation is ϵ_v and the radiation flux per unit area leaving one side of the disc is F_+. The factor of 2 multiplying this quantity accounts for the two sides of the disc. The radiative flux in the radial direction is

$$F = -\frac{c}{\kappa \rho} \frac{\partial (a_R T^4/3)}{\partial R}.$$

Vertical semi-thickness, emergent radiative flux and viscosity prescription

The vertically integrated pressure and internal energy are

$$\Pi = (\mathcal{R}/\mu)\Sigma T + 2 H a_R T^4/3$$
$$E = (3\mathcal{R}/2\mu)\Sigma T + 2 H a_R T^4.$$

The vertical semi-thickness and emergent radiation flux are

$$H = \sqrt{\frac{2\sqrt{2}E}{3\Sigma\Omega^2}} \qquad F_+ = \frac{2 a_R c T^4}{(3\kappa\Sigma + 4/3)}.$$

Viscosity is incorporated through adopting the standard α parametrization of Shakura & Sunyaev (1973). Using this, the kinematic viscosity is given by

$$\nu = \alpha P/(R\rho|\mathrm{d}\Omega/\mathrm{d}R|) \text{ leading to } \langle\nu\rangle = \alpha\Pi/(R\Sigma|\mathrm{d}\Omega/\mathrm{d}R|),$$

where $\langle\nu\rangle$ is the density-weighted vertically averaged viscosity.

Simulation set up

Simulations were performed over the radial domain [R_{in}, R_{out}] with $R_{in} = 3.0388 \times 10^{12}$ cm and $R_{out} = 1.484012$ ***X10^{14} cm.*** We have employed $N_g = 768$ equally spaced grid points and checked convergence using twice as many ($N_g = 1536$).

For a star of 1 $M_\odot$, the pericentre distance for a penetration factor B_p is given by $R_p = 7 \times 10^{12}(M/(10^6 M_\odot))^{1/3}/B_p$ For our simulations, we take $M = 10^6 M_\odot$ and $B_p = 14/9$. Then the pericentre distance is 4.5×10^{12} cm and the minimum return time is $P_{\rm min} = 3.5 \times 10^6 B_{\rm p}^{-3}$ s $= 9.30 \times 10^5$ s.

We adopt the following simplified prescription for the accretion rate $\dot{M}_{\rm S}$ from the stream generated by the tidally disrupted star. Setting $t = 0$ to be the time of pericentre passage, we take $\dot{M}_{\rm S} = 0$ for $t < P_{\rm min}$. For $t > P_{\rm min}$, we set $\dot{M}_{\rm S} = 7.17 \times 10^{26} (t/P_{\rm min})^{-5/3}$ g s^{-1}. Thus, we assume a tail off $\propto t^{-5/3}$

Mass and energy input from stream

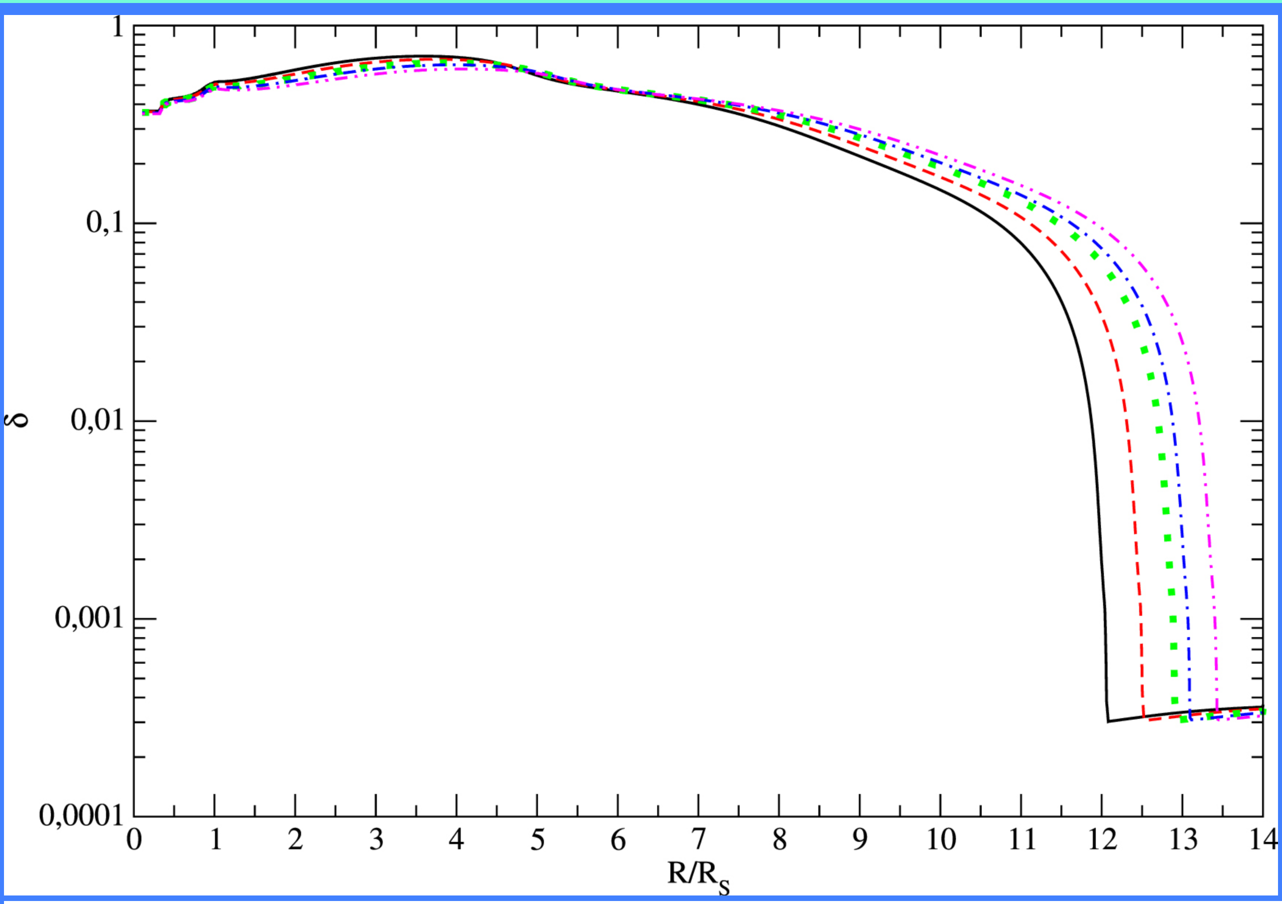
When mass from the stream enters the disc, energy is dissipated. Assuming that the plane of the stream is only slightly inclined to that of the disc, the kinetic energy per unit mass associated with radial motion available to be dissipated is $GM/(2R_S)$.

Depending on details of the circularization process, a part of this is radiated away directly and a part is converted to excess internal energy of the disc. This energy is input along with and in the same way as the mass input in this way S_E is determined.

For simplicity, we have assumed this fraction to be f_{st} = 50 per cent for the

The steam commences to input mass at $t = P_{min}$ into an initial disc. This was specified to have a low mass of 0.011 $M_\odot$ as compared to the total to be input

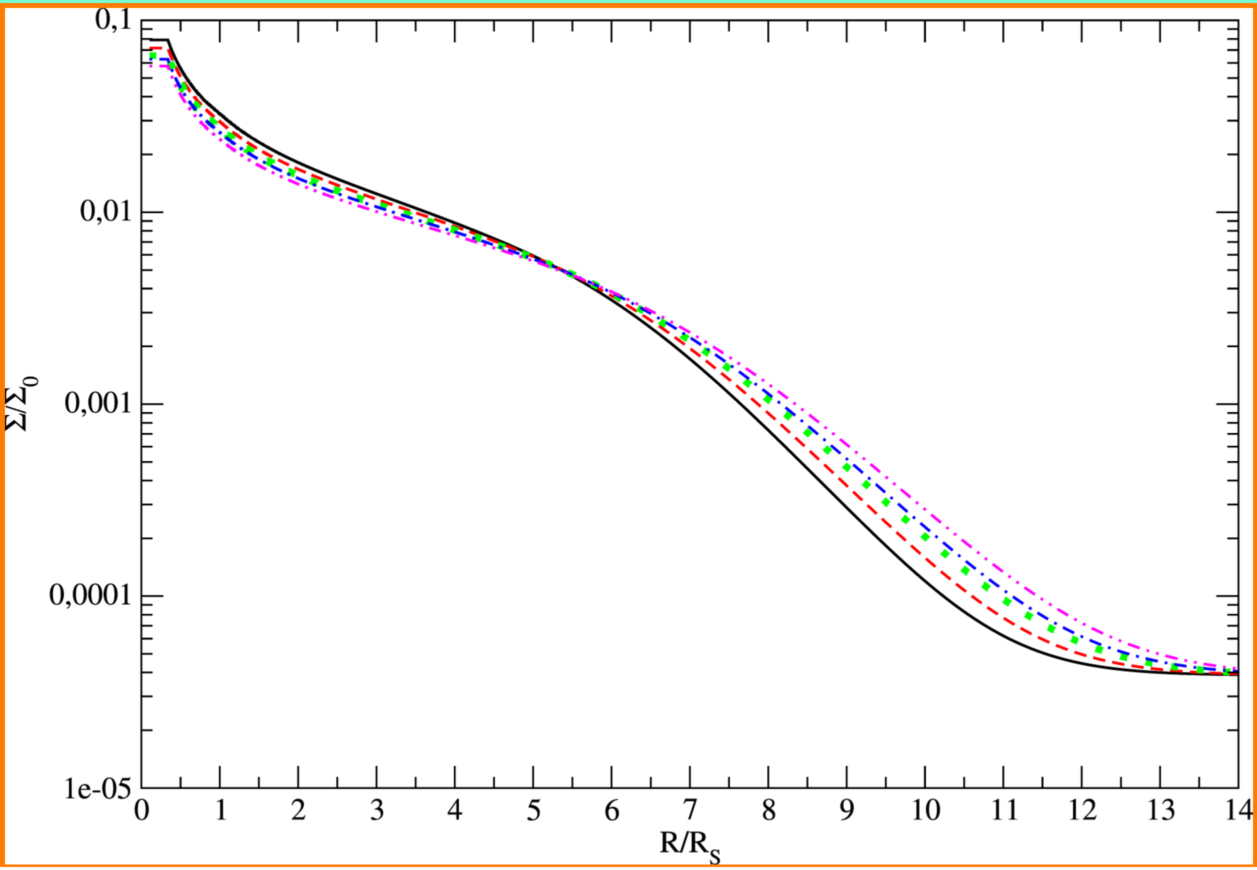
Outward propagating front for model with α =0.01 at early times



Characteristic front propagation speed on the order of $\alpha\delta R\Omega$

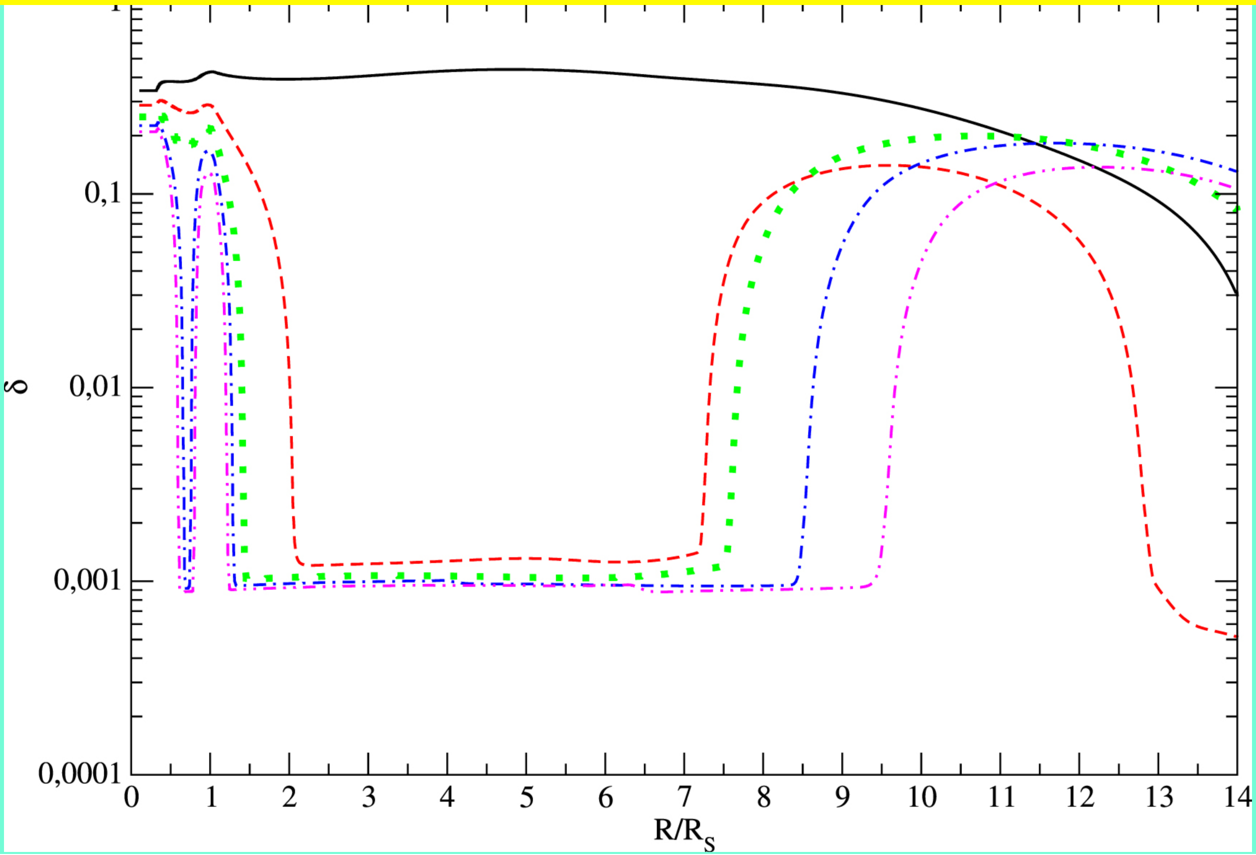
The dependence of disc aspect ratio δ on radius calculated for the low viscosity run with $\alpha = 0.01$. Solid, dashed, dotted, dot-dashed, and dot-dot-dashed curves correspond to $t/P_{\min} \approx 11, 12, 13, 14, 15$, respectively.

Surface density distribution for model with α =0.01 at early times



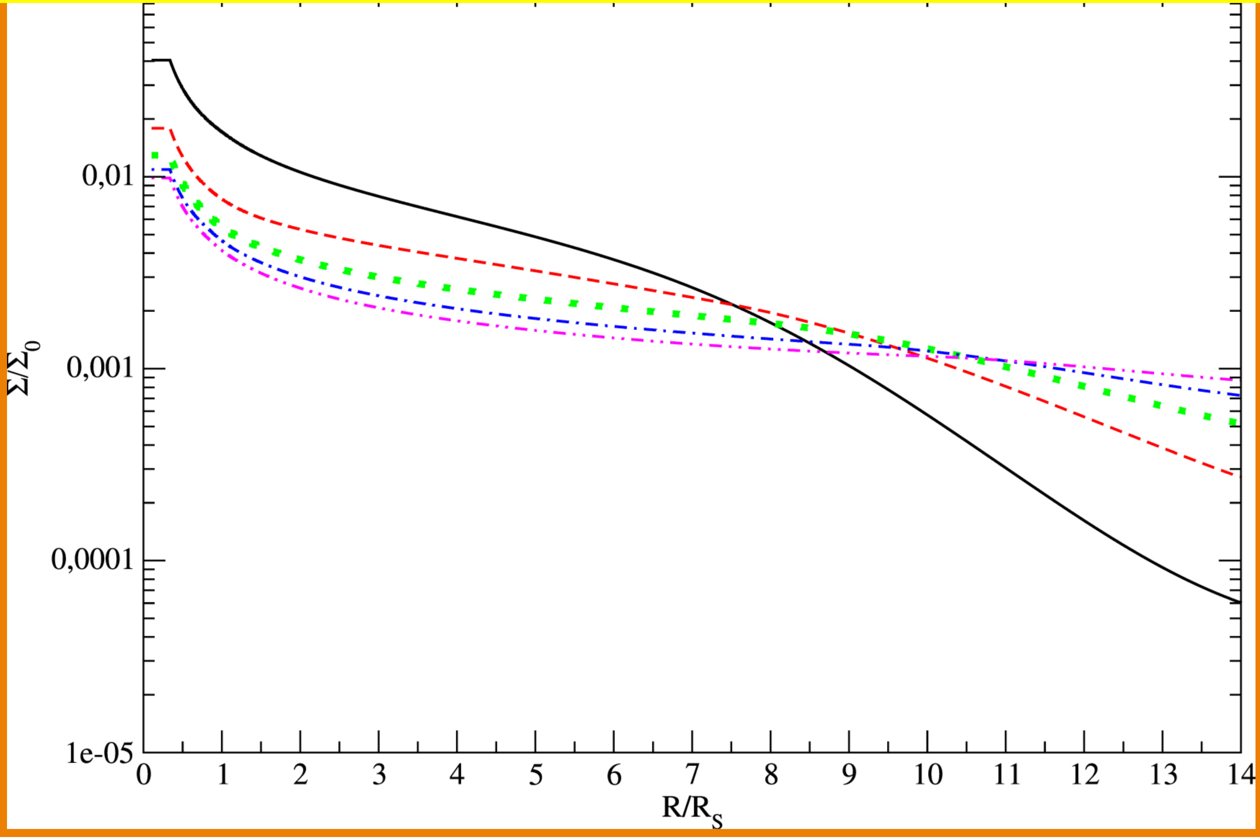
The disc's surface density in units of $m/(2\pi R_S^2)$.
Solid, dashed, dotted, dot-dashed and dot-dot-dashed curves are for t/P_{min} =11, 12, 13, 14, and 15 respectively

Semi-thickness for model with α =0.01 at later times



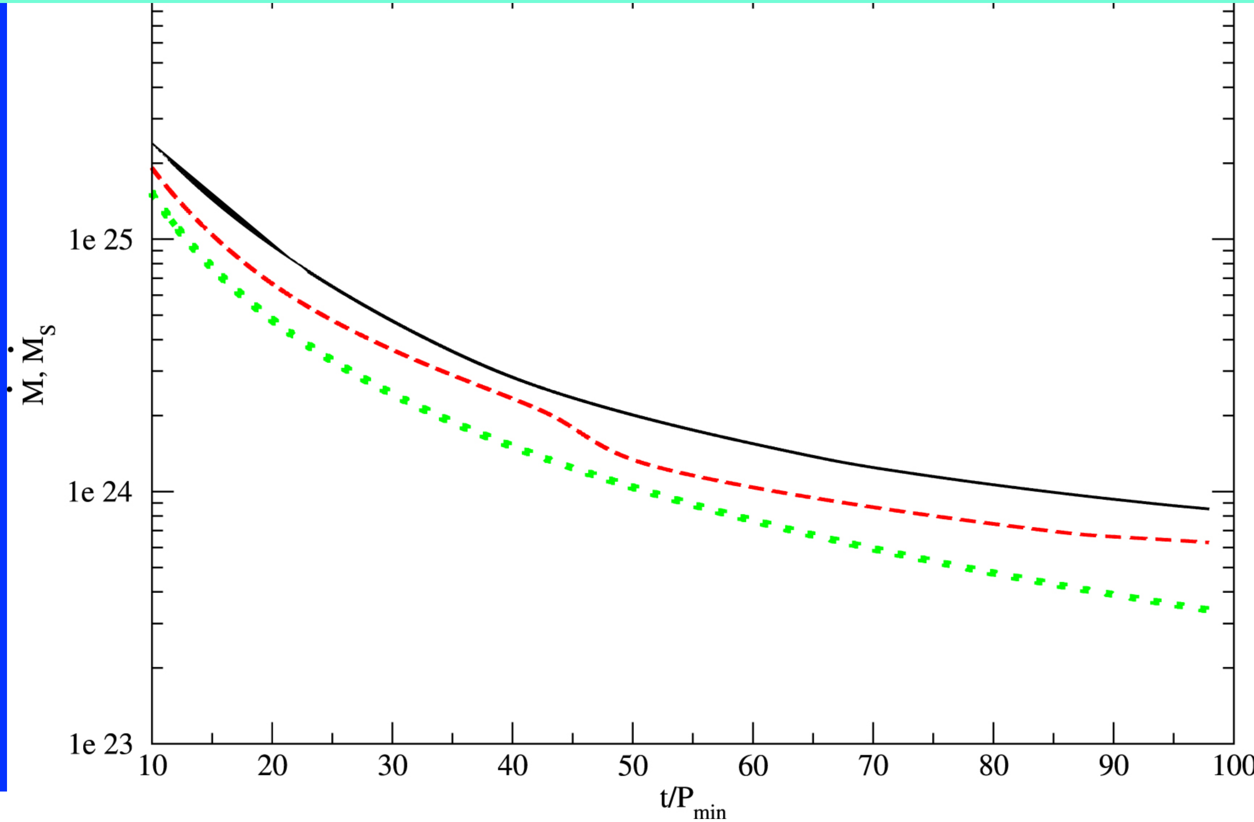
The disc's aspect ratio at later times − resurgent thermal instability at larger radii. Solid, dashed, dotted, dot-dashed and dot-dot-dashed curves are for t/P_{min} =20, 40, 60, 80, and 100 respectively

Surface density distribution for model with α =0.01 at later times



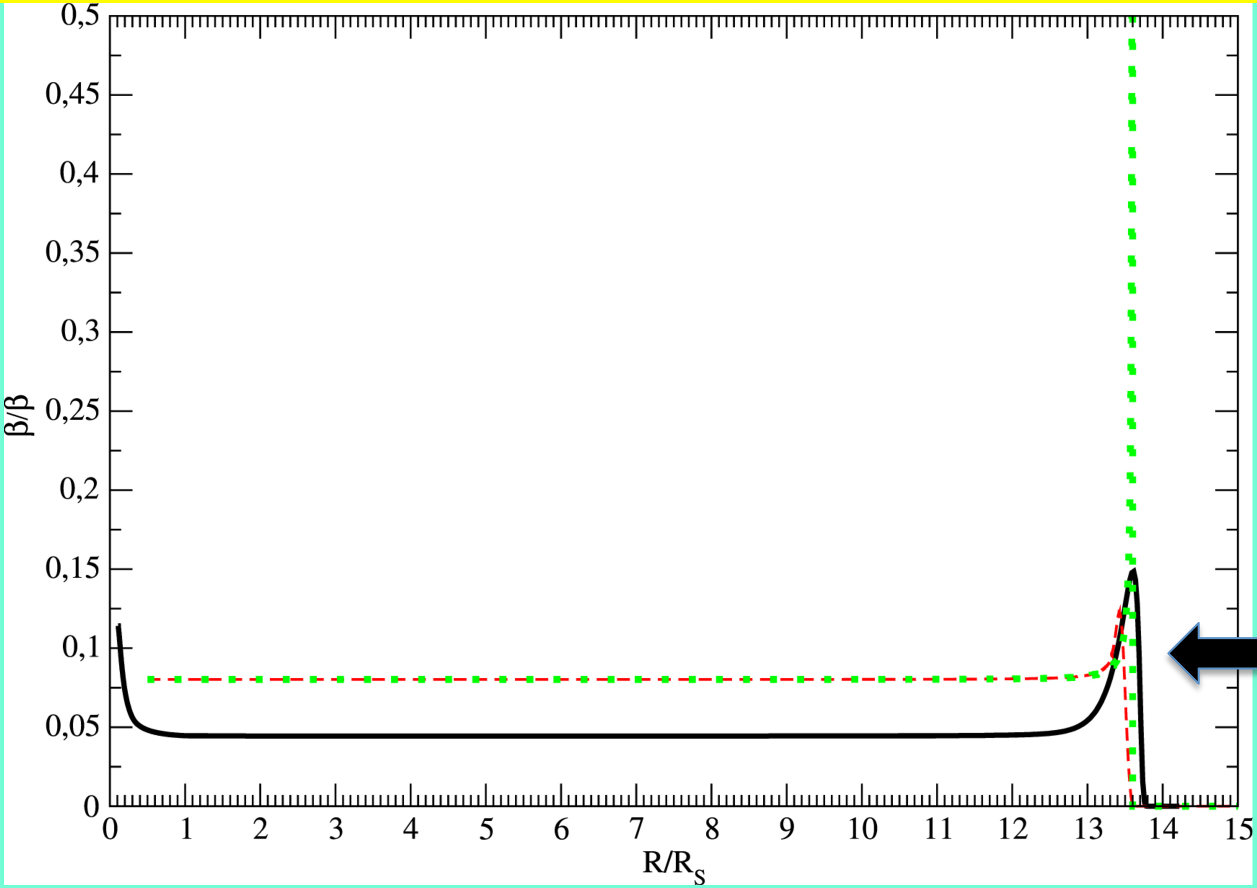
The disc's surface density in units of $m/(2\pi R_S^2)$ at later times. Solid, dashed, dotted, dot-dashed and dot-dot-dashed curves are for t/P_{min} = 20, 40, 60, 80, and 100 respectively

Accretion rates for models with α =0.1 and α =0.01



Accretion rates in g/s as functions of time are shown as solid and dashed curve for $\alpha = 0.01$ and $\alpha = 0.1$, respectively. The dotted curve shows the mass flow in the stream

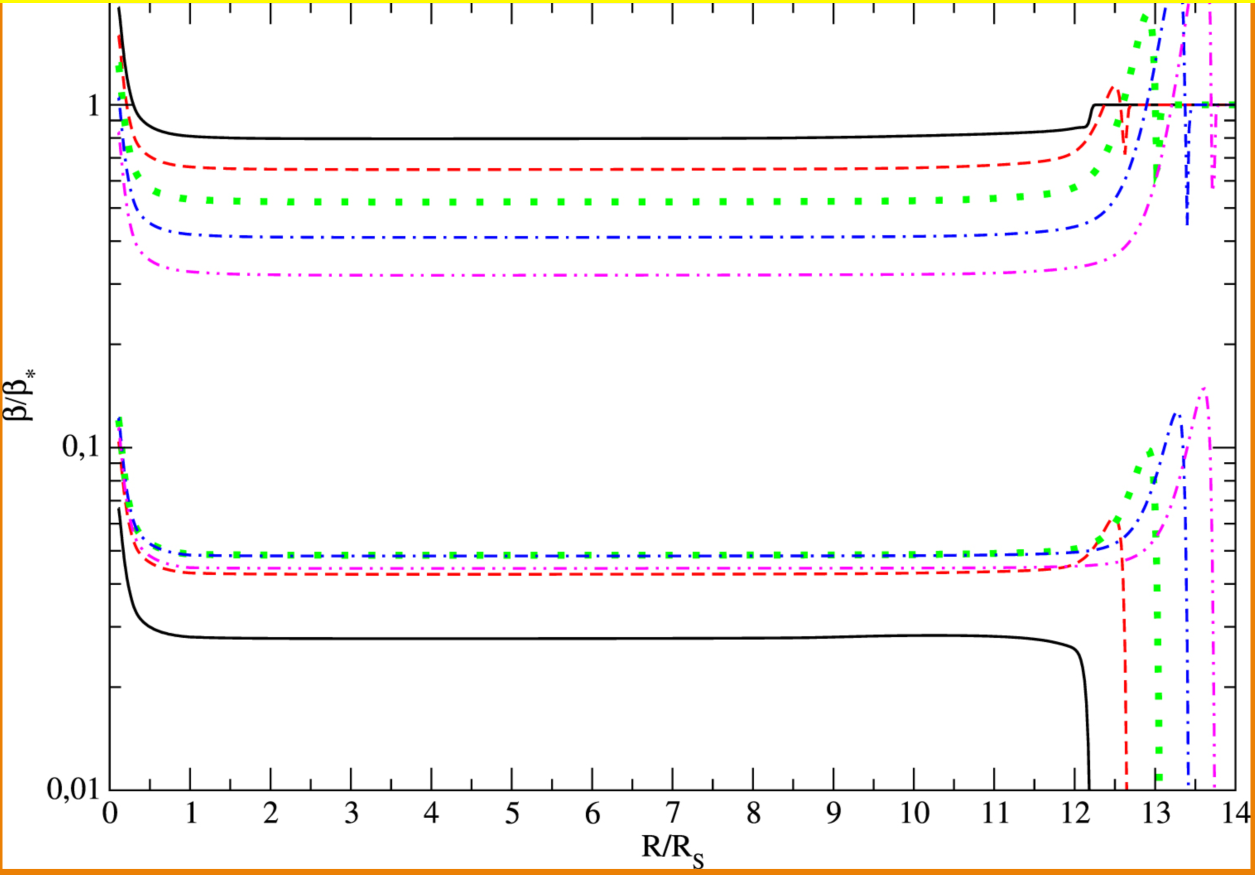
Dependence of inclination for model with α =0.01 on R after some front propagation



Pile up of bending waves and disc tearing

A comparison of the dependence of β on R found from numerical simulation solving (★) (Solid curve).
The dashed and dotted curves are from semi-analytic modeling.
Note that additional viscosity is applied in the vicinity of the transition region between a high and low state.

Form of the inclination as the front evolves



The dependence of β on R found calculated at early moments of time when a transition front was propagating outwards. (Solid curve).
This is flat until the transition is reached giving rise to an apparent tearing.
The lower /upper curves are for initial values of β taken to be zero/the value corresponding to the stream respectively .
Note that additional viscosity is applied in the vicinity of the transition region between a high and low state.

Discussion

We have studied the evolution of disks formed by the tidal disruption of a star by a black hole when there is a misalignment between the black hole equatorial plane and the orbital plane of the stellar orbit .
That inclination angle being and a non-trivial function of time and radius could have implications for the time evolution of the observed disc luminosity.
This could allow tests of different modes of the accretion process and provide information on the black hole mass and angular momentum. Moreover, a thick inclined disk respect as expected at 'early' evolution times may produces a jet directed perpendicular its plane and determine the evolution of the jet luminosity.