OSCILLATIONS OF TORI AND DISKS
IN GR HYDRO SIMULATIONS

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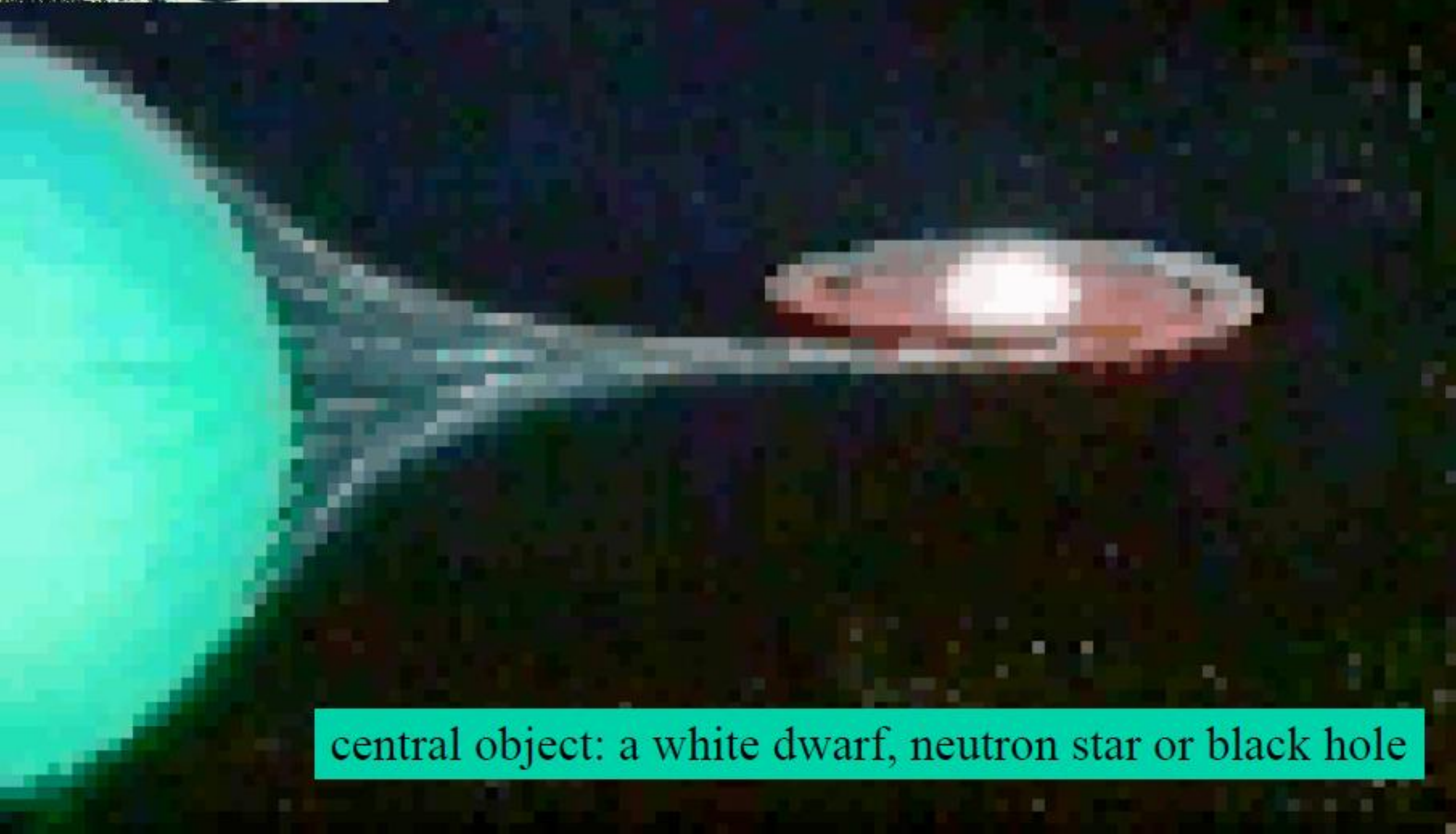
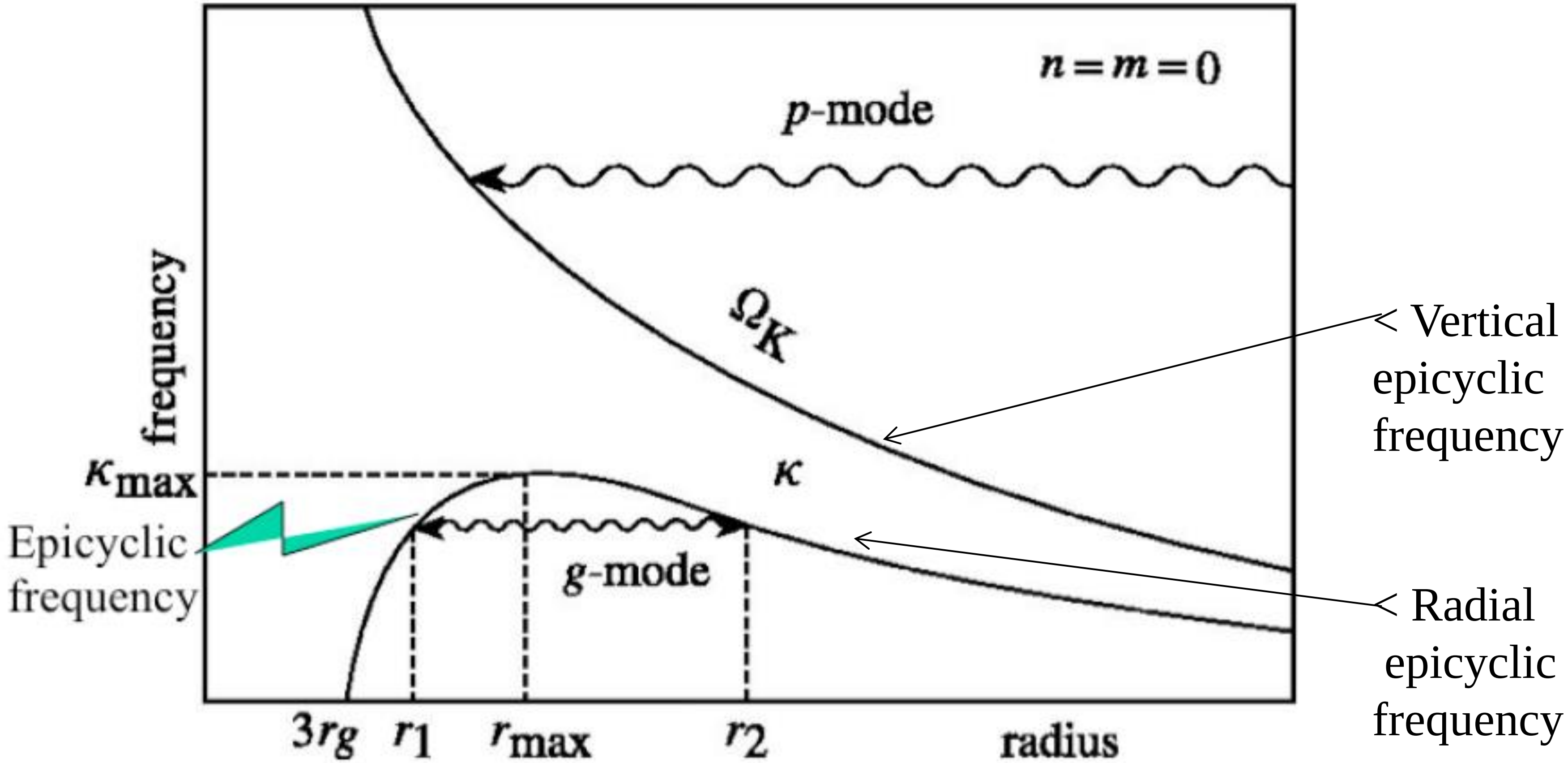
Cosmos++ simulations by Bhupendra Mishra

Slim disk workshop CAMK 20.10.2018


central object: a white dwarf, neutron star or black hole

Okazaki, Atsuo T.; Kato, Shoji; Fukue, Jun: PASJ 39 (1987) 457
Global trapped oscillations of relativistic accretion disks

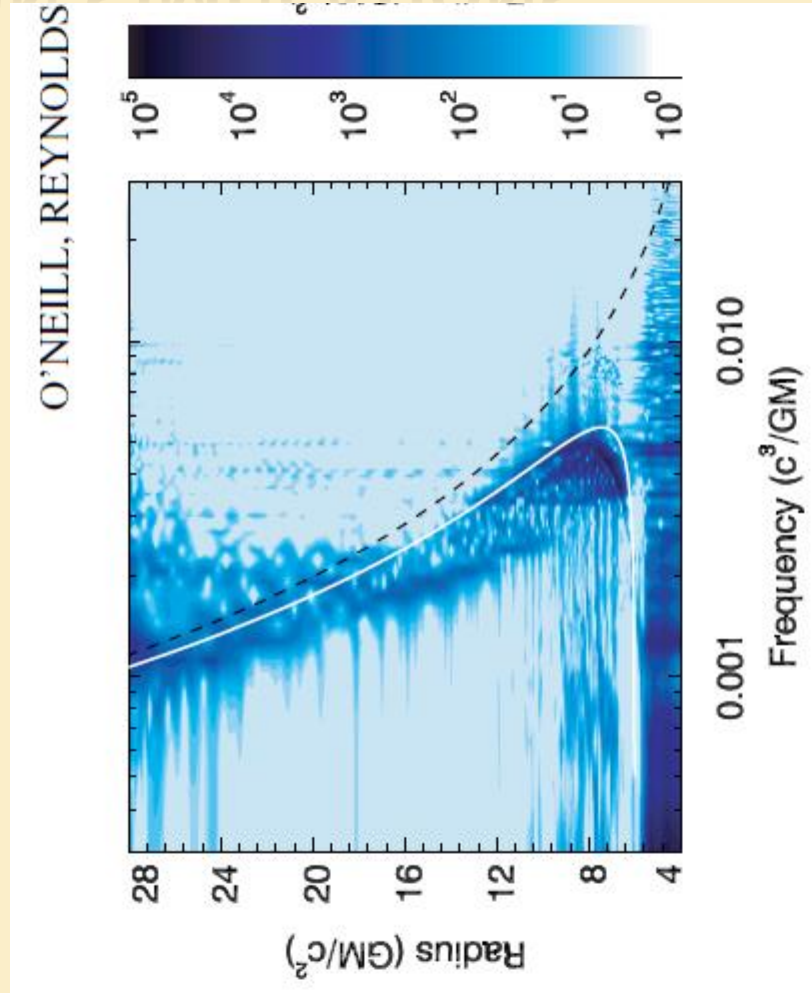


Kato 2001 ⇓⇒ Diskoseismology of Nowak & Wagoner …

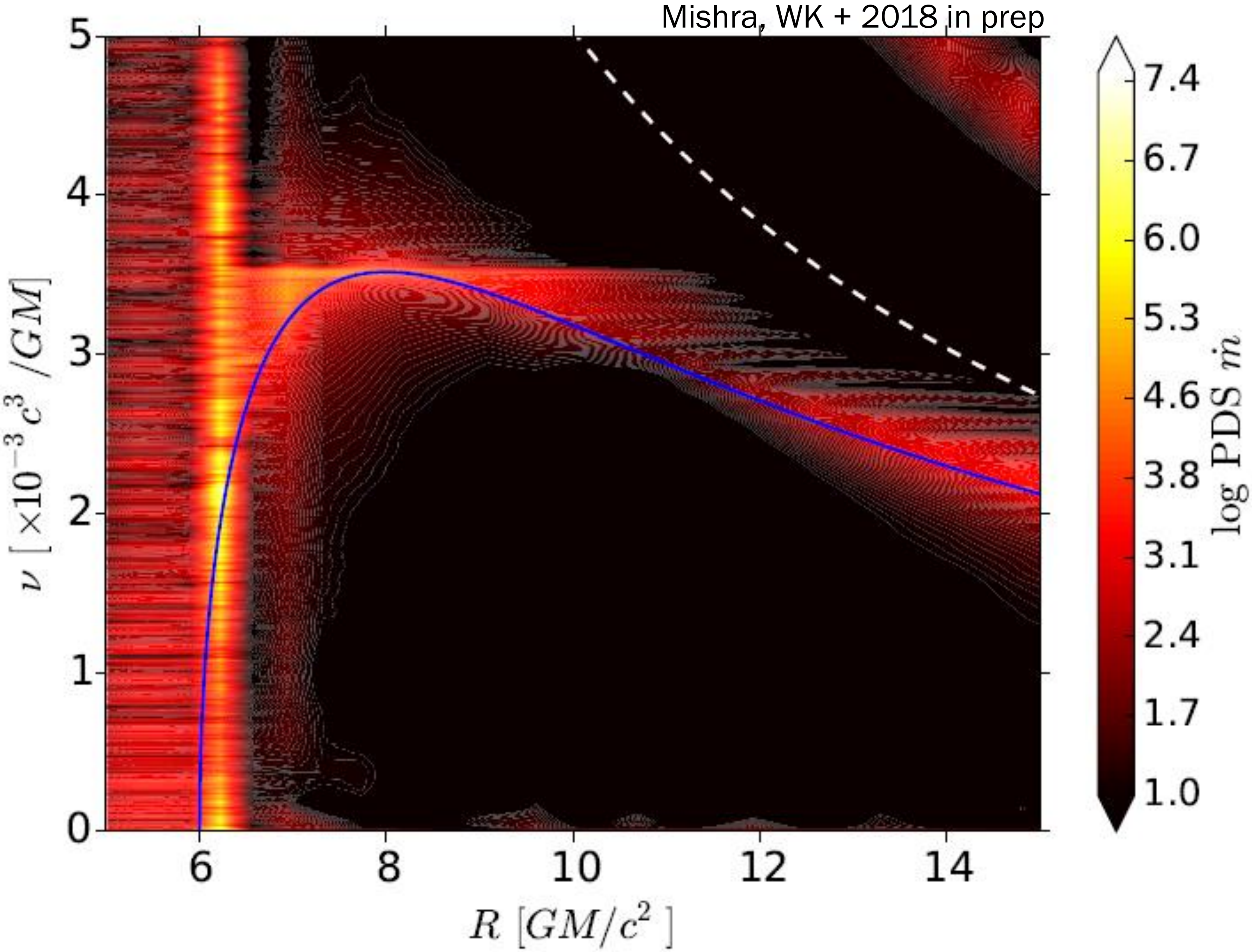
Wagoner 1999, Phys Rep

THIN-DISK GLOBAL MODES IN SIMULATIONS

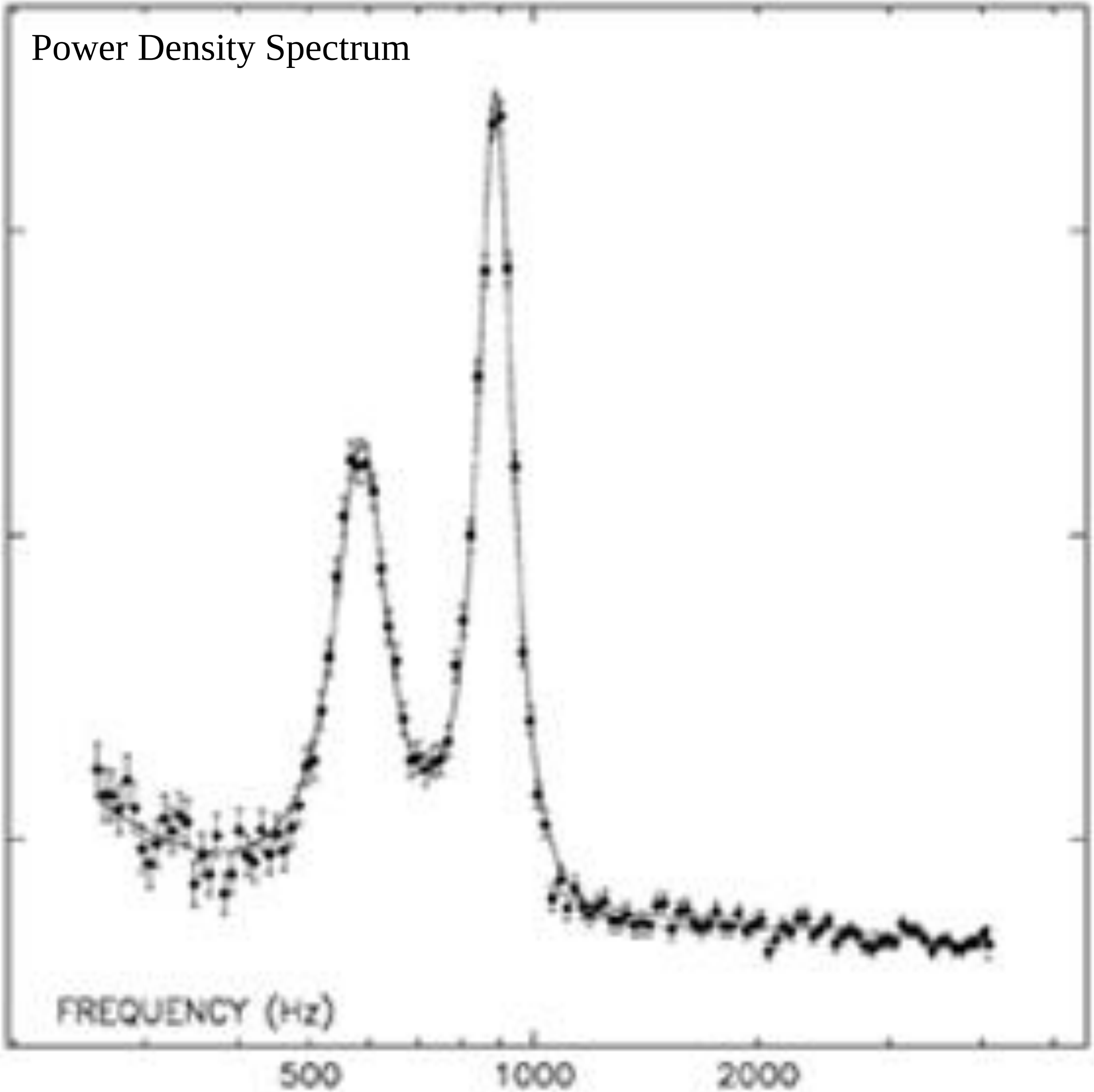
- *g*-modes reported for disks
  in Paczyński's pseudopotential
  Newtonian hydro simulations
  (Reynolds + 2009, O'Neill + 2009).
- Not seen in MHD (*ibid*.):
  MRI "destroys" *g*-modes.
- Our **radiative GR** simulations of
  accretion disks (Mishra + 2018)
  confirm this picture.



- However, QPOs of different origin reported in MHD simulations of **THICK** flows (Y. Kato, Matsumoto).



Mishra, WK + 2018 in prep
5
4
3
2
1
0
6
8
10
12
14
$\nu\ [\times 10^{-3}\ c^3\ /GM]$
$R\ [GM/c^2\ ]$
7.4
6.7
6.0
5.3
4.6
3.8
3.1
2.4
1.7
1.0
$\log \mathrm{PDS}\ \dot{m}$



High Frequency
Quasi Periodic
Oscillations
in Sco X-1
(NS candidate)

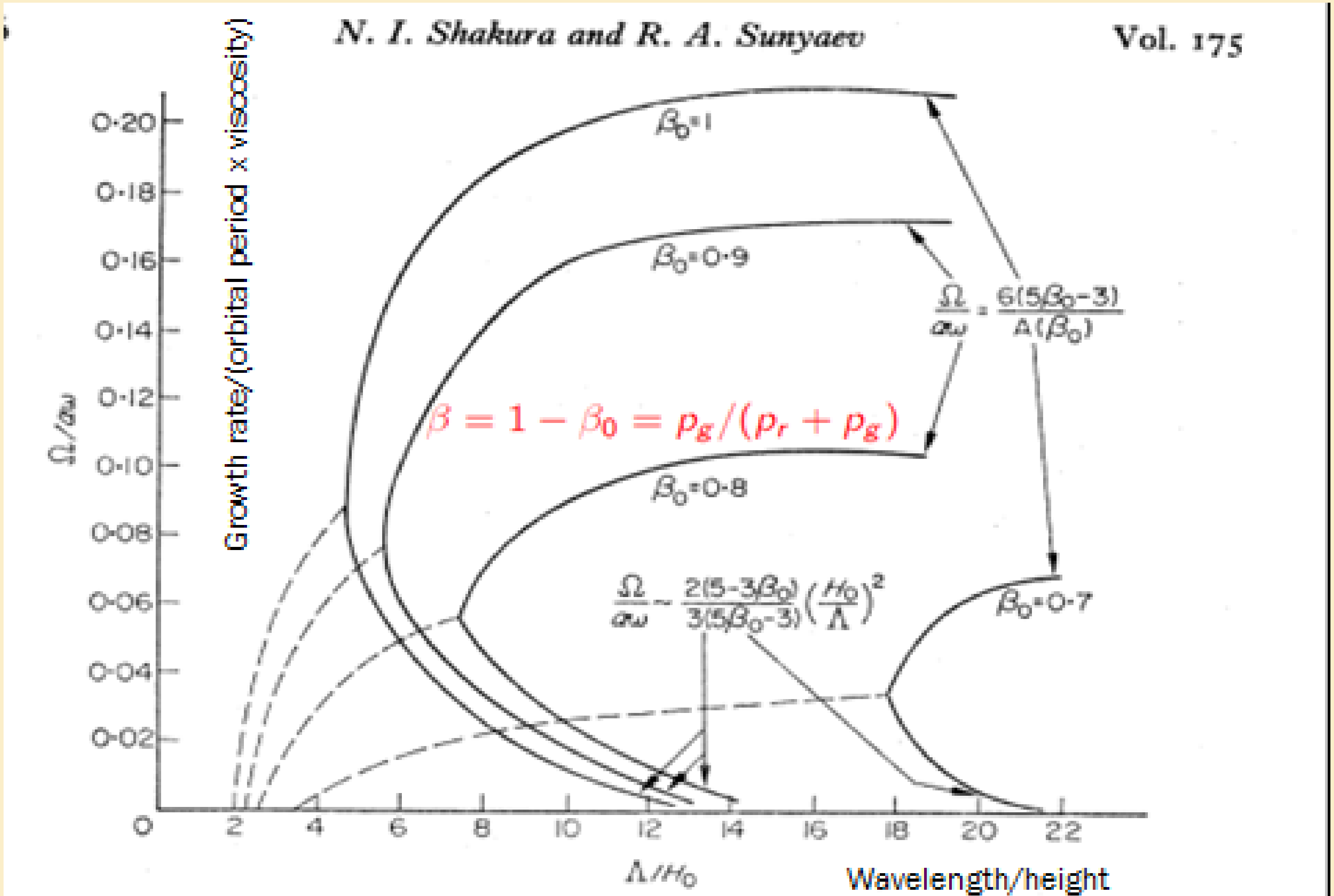
kHz => kilometers from NS/BH

(Rossi XTE observation)

van der Klis et al. 1996

RADIATION-PRESSURE DOMINATED THIN ACCRETION DISKS UNSTABLE

- α-disks Sunyaev & Shakura 1975, Shakura & Sunyaev 1976
- Confirmed by shearing box MRI simulations (Jiang+2013)

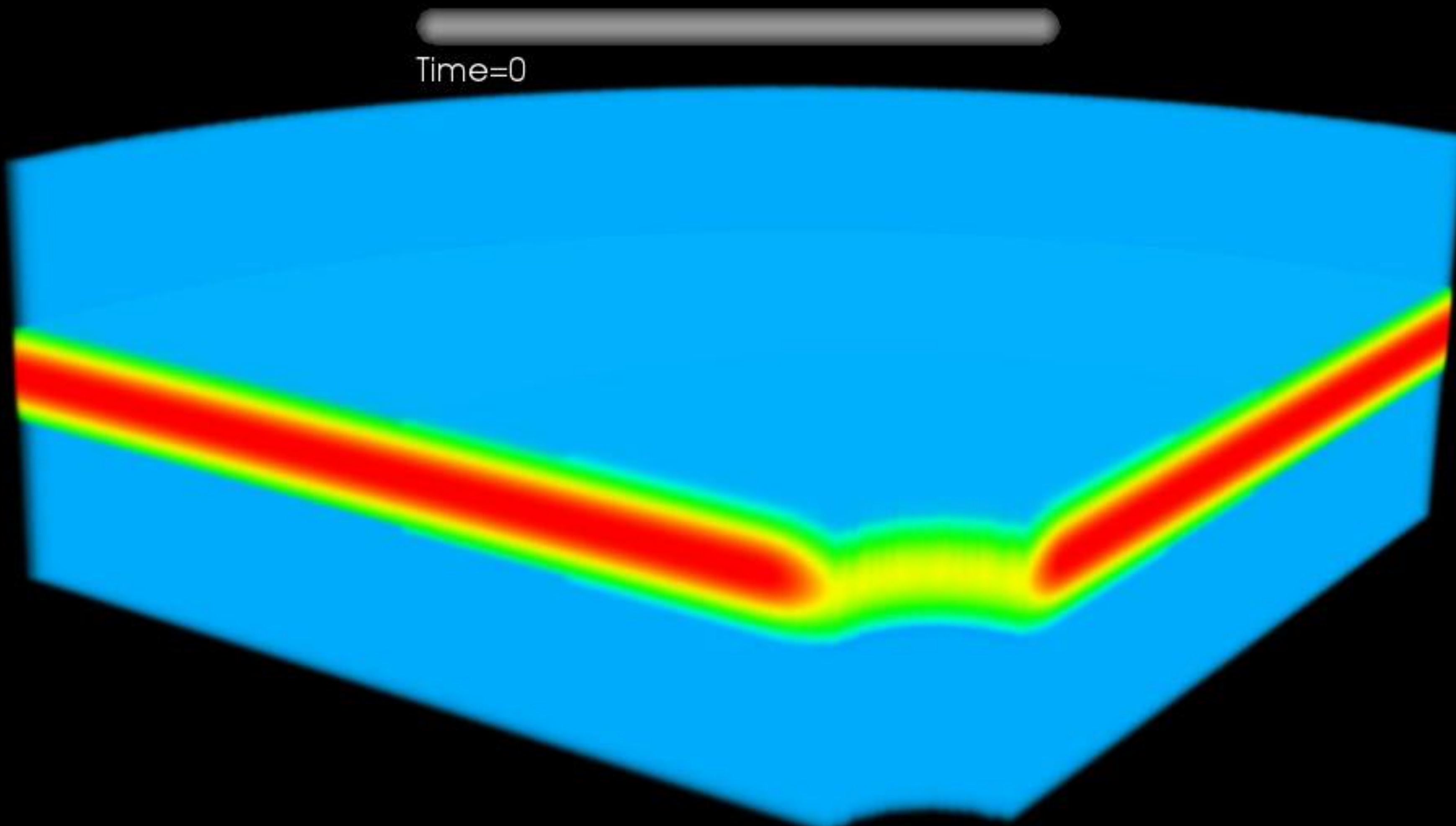


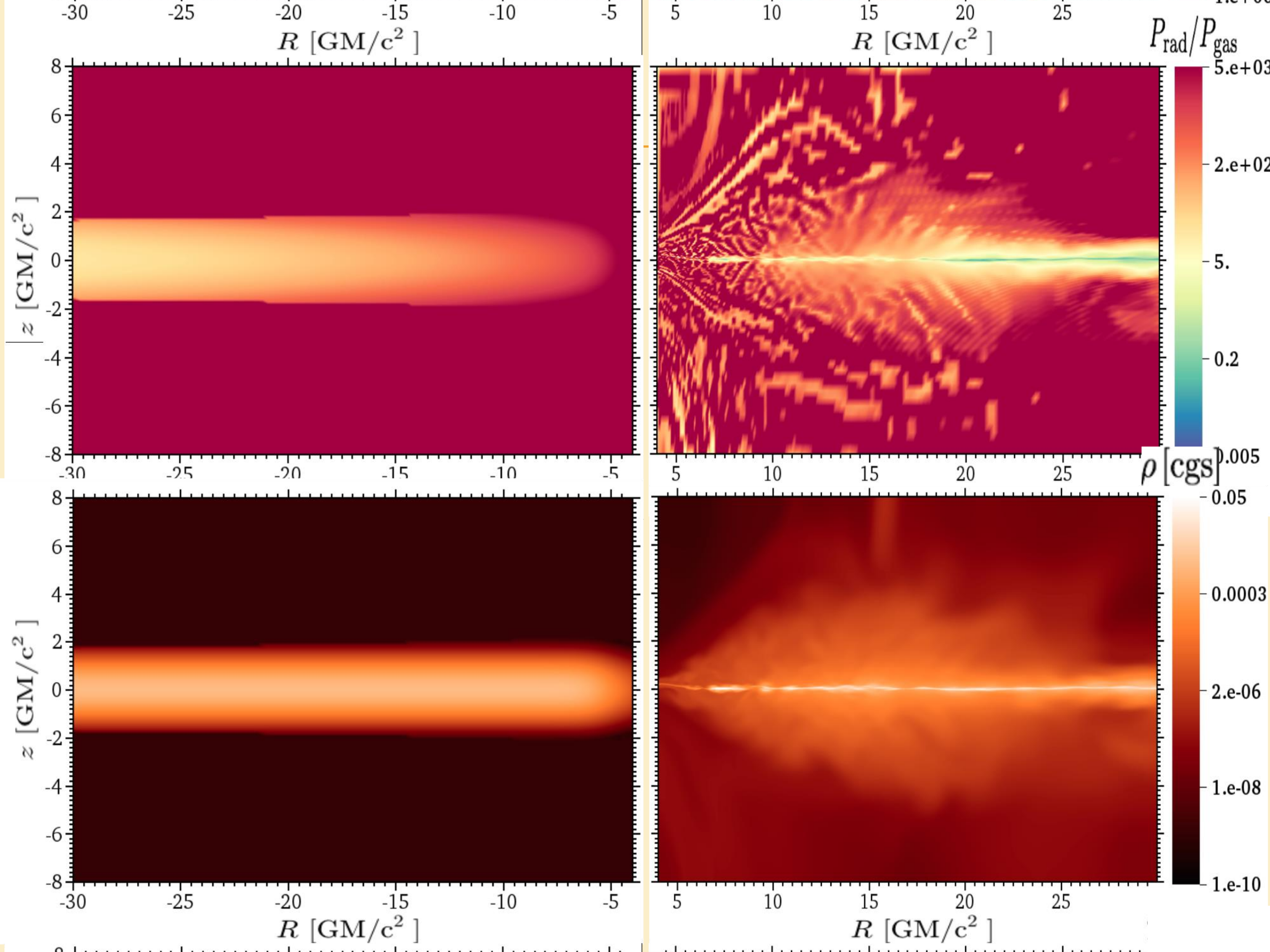
GLOBAL RADIATIVE GRMHD THIN-DISK SIMULATIONS

Brief report on first global radiative GR simulations of thin disks:
Gas-*P* dominated stable in our GRRMHD.

Radiation pressure dominated thin accretion disk unstable:
(advection and magnetic fields may stabilize slim disks)

- Our global radiative GRMHD geometrically **thin** disk confirm Shakura-Sunyaev 1976 thermal instability (Mishra+2017).
- Previously seen in shearing-box MRI simulations (Jiang+2013)
- Same results in our GRRHD (Fragile+2018).
- Evidence for Lightman-Eardley instability? (*ibid.)*


Time=0



R [GM/c^2]
z [GM/c^2]
$P_{\rm rad}/P_{\rm gas}$
5.e+03
2.e+02
5.
0.2
0.005
ρ [cgs]
0.05
0.0003
2.e-06
1.e-08
1.e-10

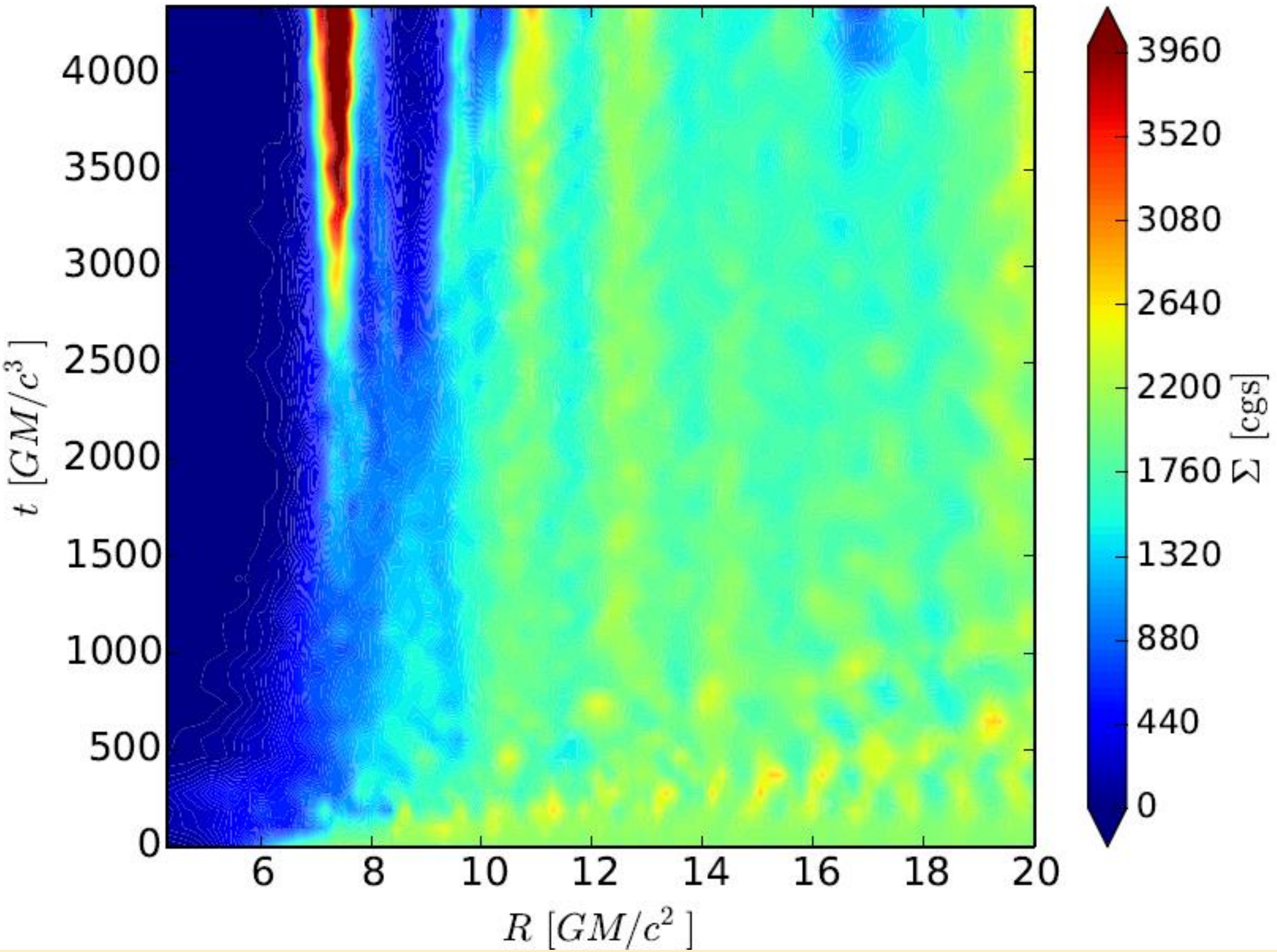
VISCOUS INSTABILITY

- Lightman-Eardley 1974

  combine 2 eom => diffusion equation
- Radiation pressure dominated disk

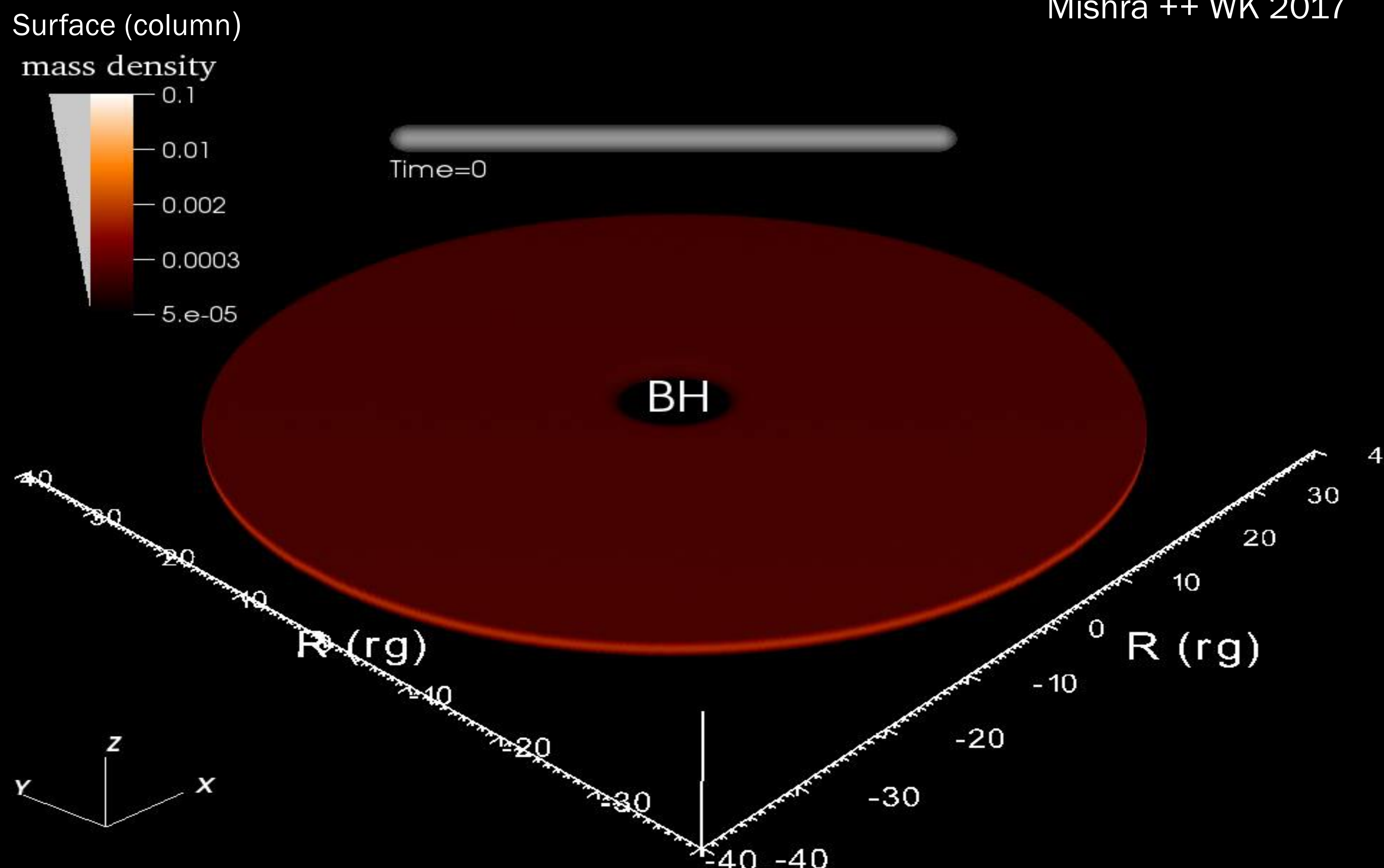
  => Negative diffusion coefficient !

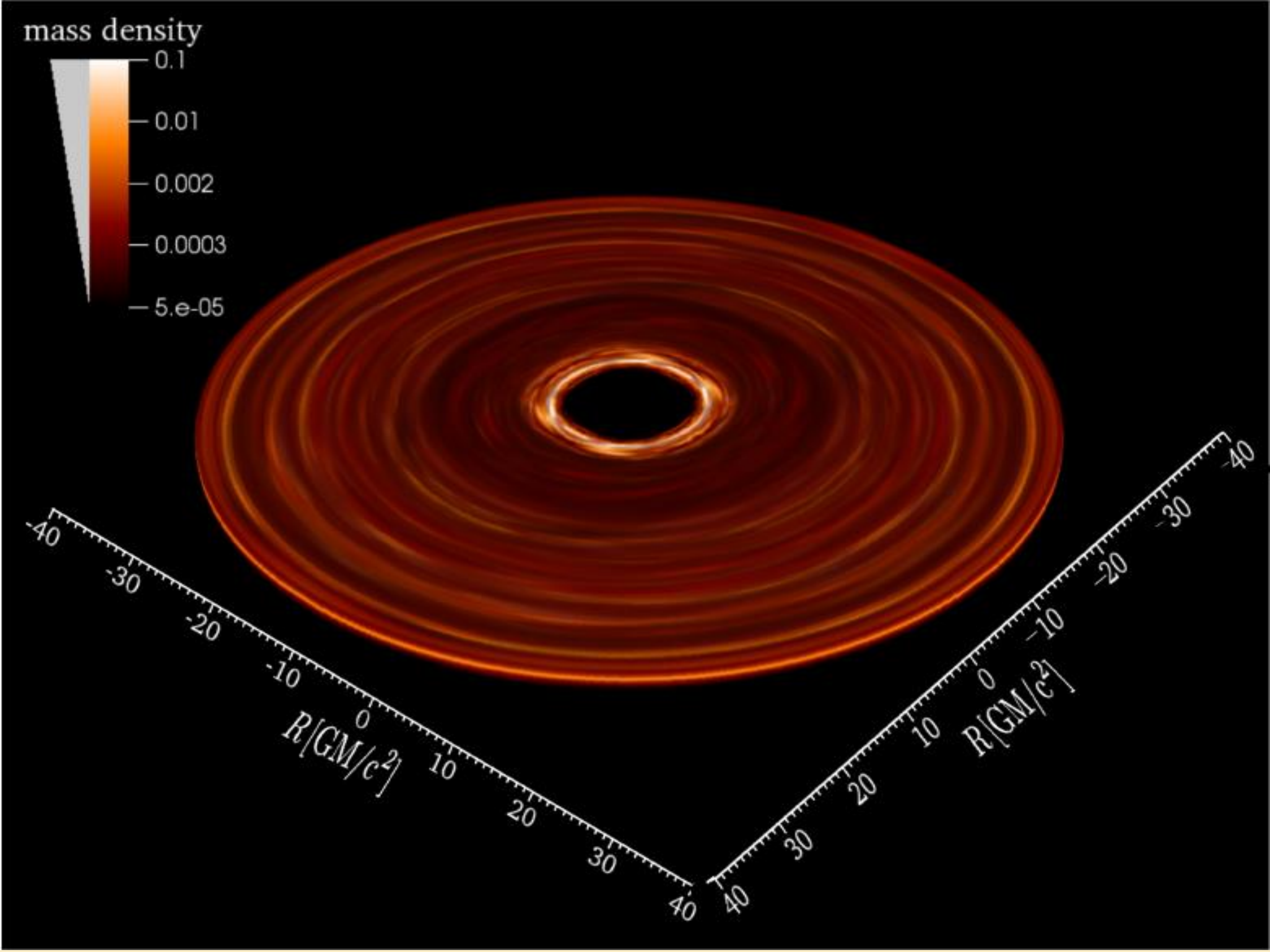
  => rings (tori) form in the disk.
- Seen in our simulation for the first time in 43 y.



4000
3500
3000
2500
2000
1500
1000
500
0
$t\ [GM/c^3\ ]$
6
8
10
12
14
16
18
20
$R\ [GM/c^2\ ]$
3960
3520
3080
2640
2200
1760
1320
880
440
0
Σ [cgs]

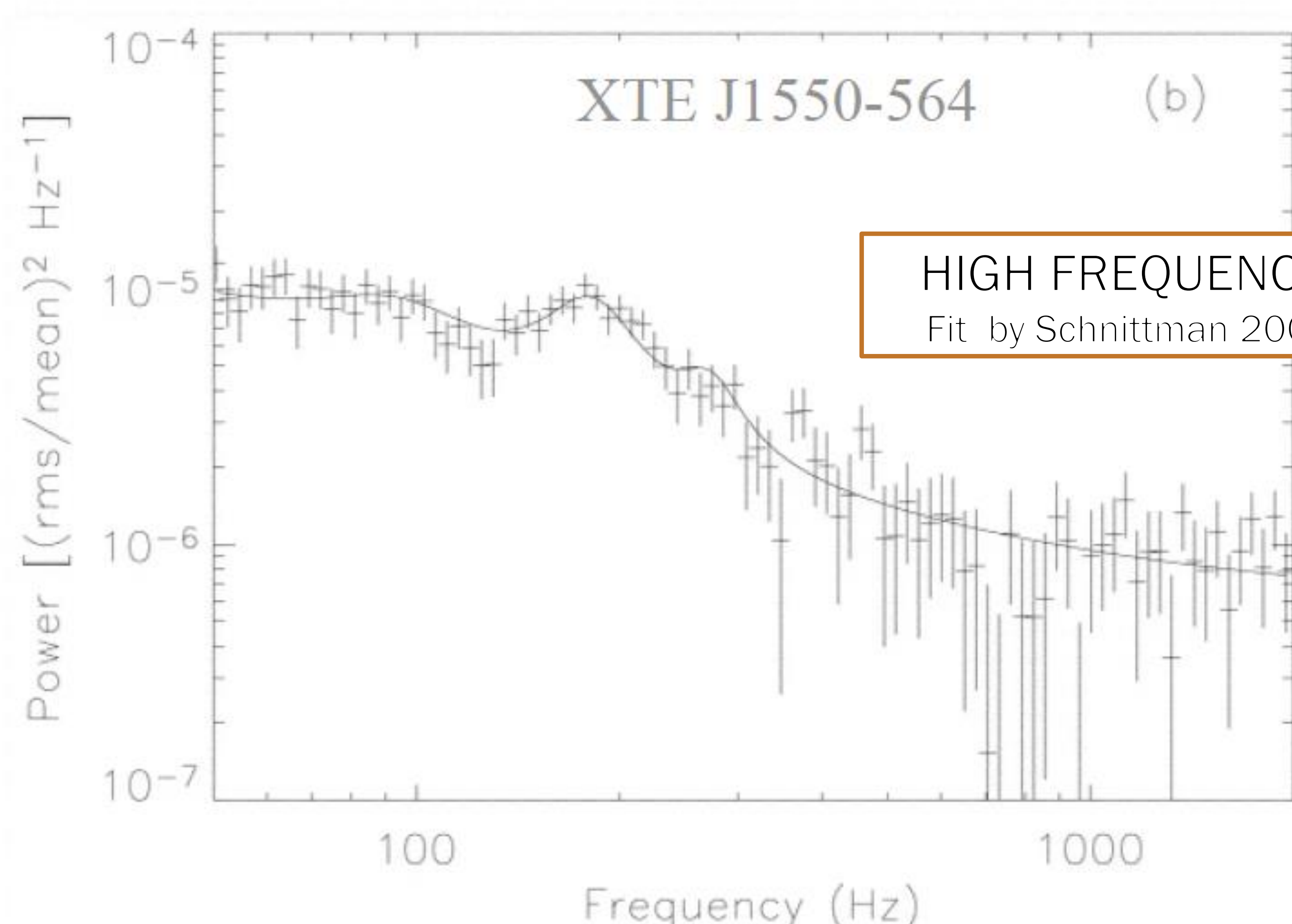
Mishra ++ WK 2017





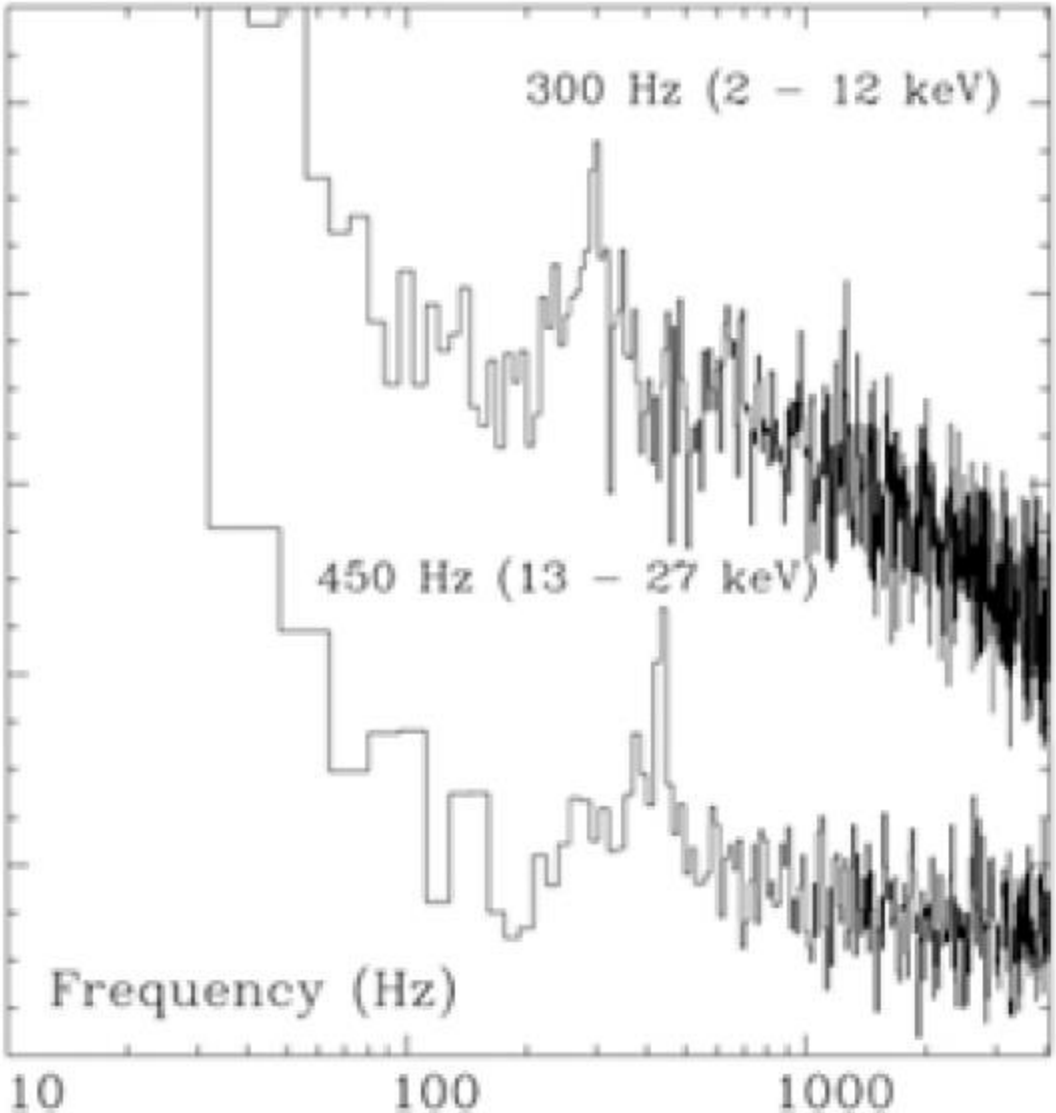
mass density
0.1
0.01
0.002
0.0003
5.e-05
-40
-30
-20
-10
0
10
20
30
40
R[GM/c^2]
-40
-30
-20
-10
0
10
20
30
40
R[GM/c^2]

Normal modes of similar tori have been invoked to explain the observed high frequencies in X-ray binaries (kHz in NS, hHz in BH), e.g., Bursa et al. 2004.



HIGH FREQUENCY QPOs

Fit by Schnittman 2005, a=0.5

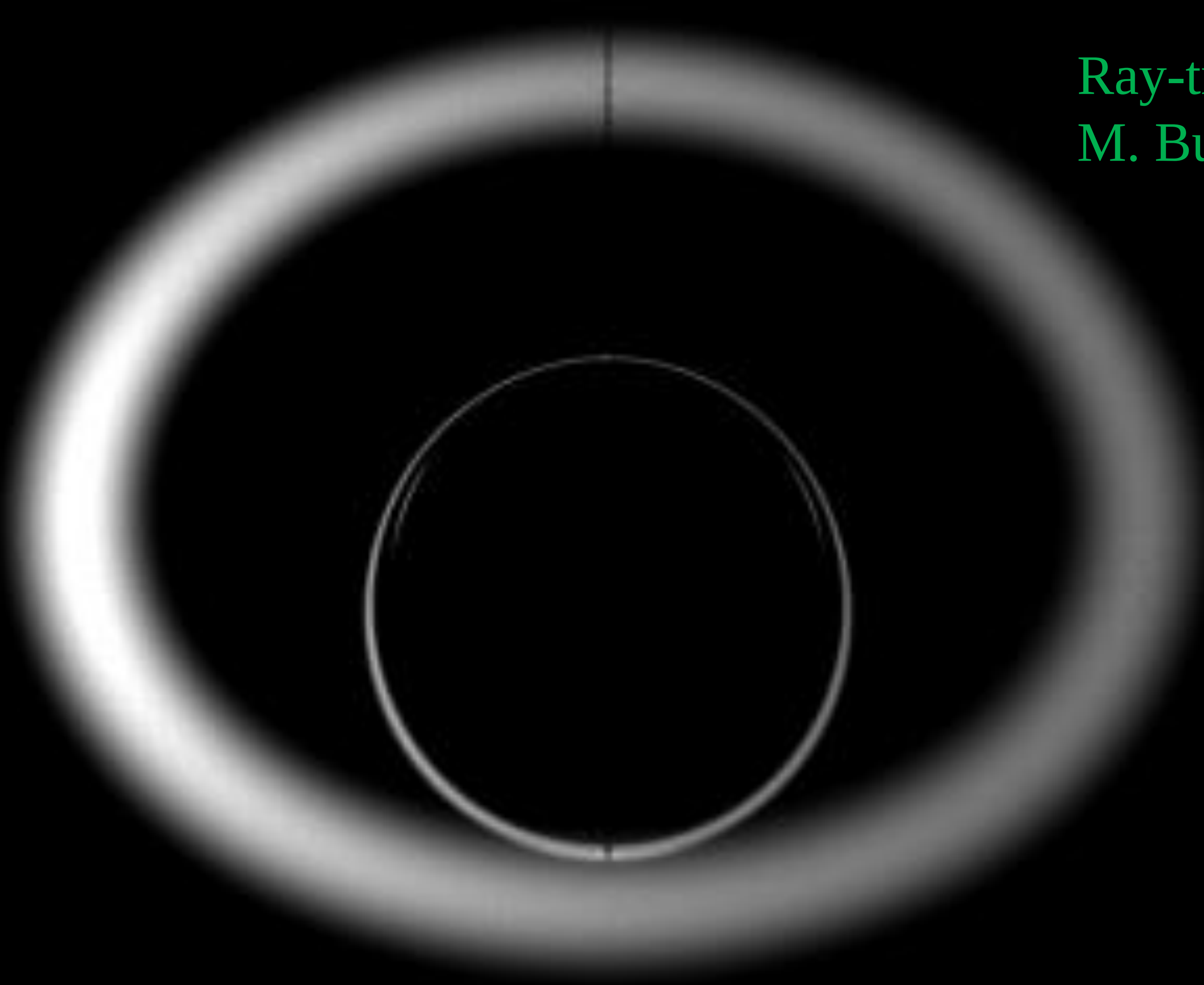


GRO J 1655-40 X-ray power spectra : discovery of second HFQPO in a BH (Strohmayer 2001).

Abramowicz & Kluźniak 2001, *A precise measurement of BH spin* (A&A):

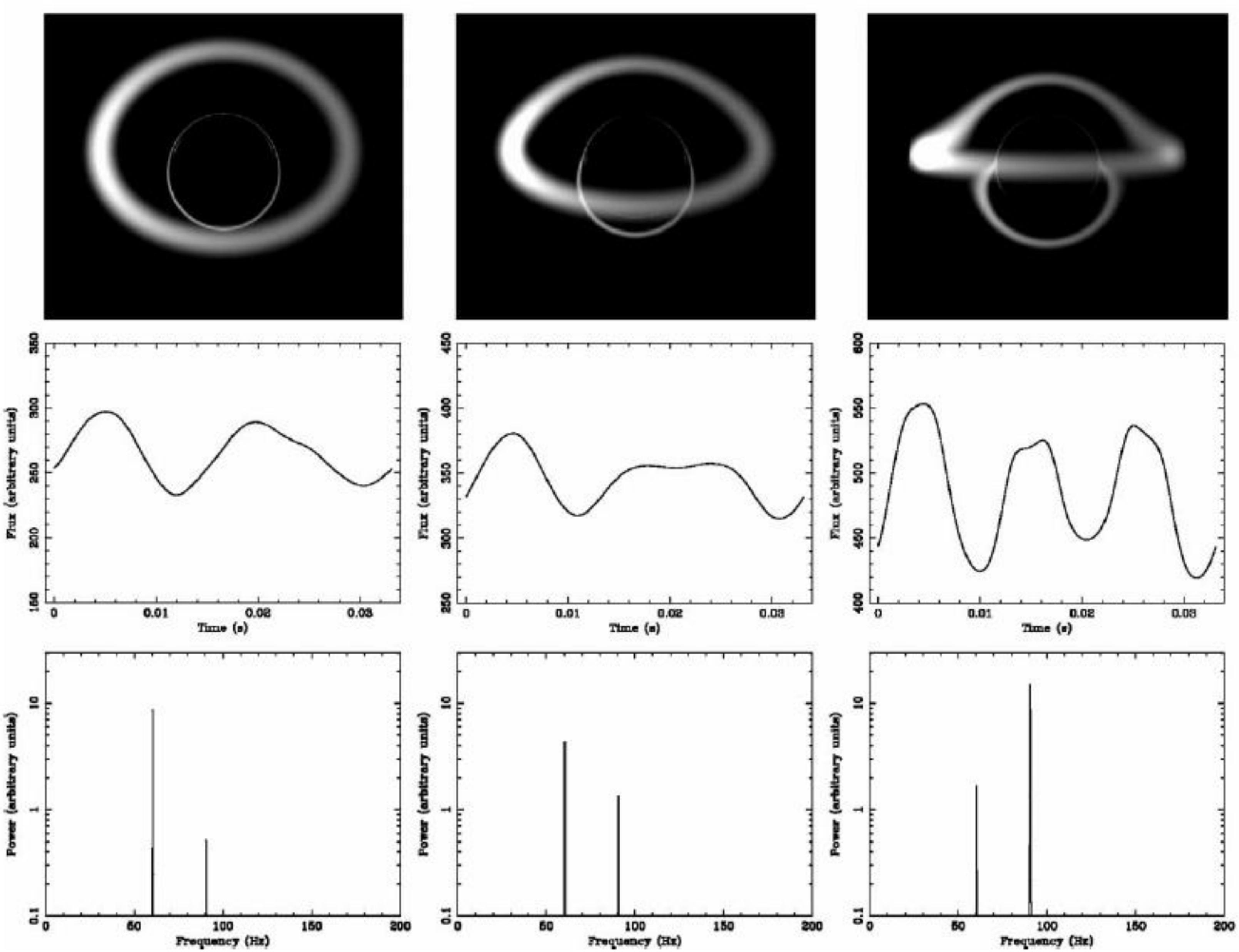
'We note that the recently discovered 450 Hz frequency in the X-ray flux of the black hole candidate GRO J1655-40 is in a 3:2 ratio to the previously known 300 Hz frequency...'

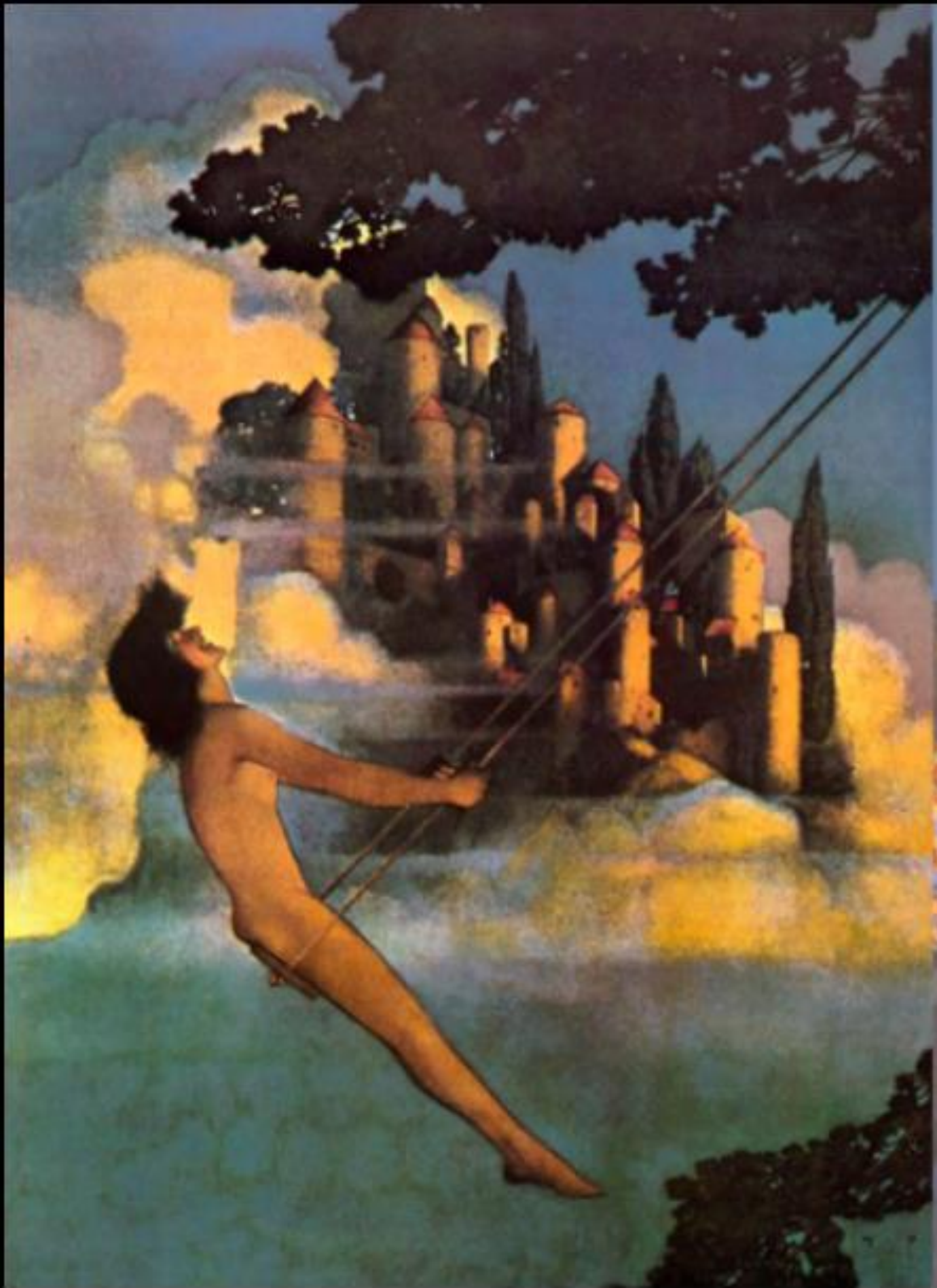
(Resonance model: WK & MAA 2000/2001)



Ray-traced by
M. Bursa

Bursa et al. 2004





Maxfield Parrish

Thunderheads $\omega^2=g/l(t)$

1943

Cover

22 1/2 x 18 inches (57.15 x 45.72 cm)

Private collection

↲

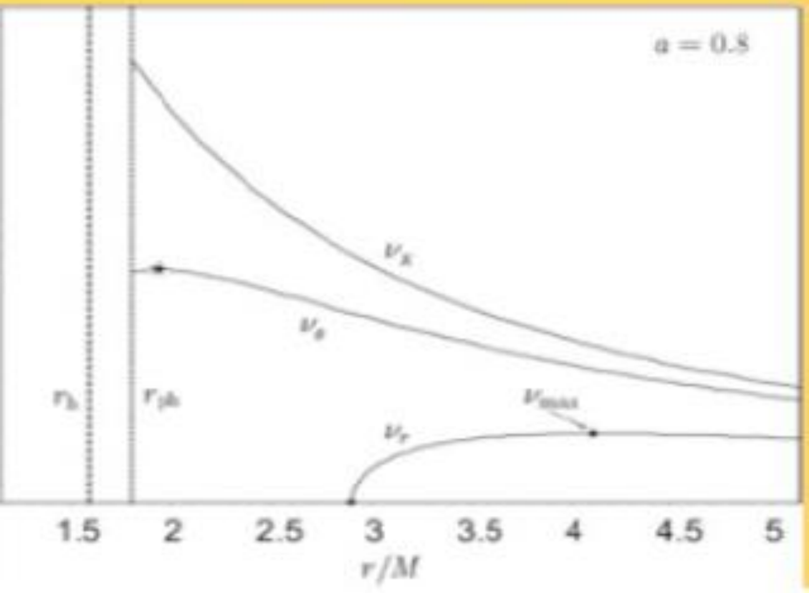
Parametric resonance: Mathieu eq.

$\ddot{U} + \omega_0^2[1 + h\cos(\omega_1 t)]\, U = 0$

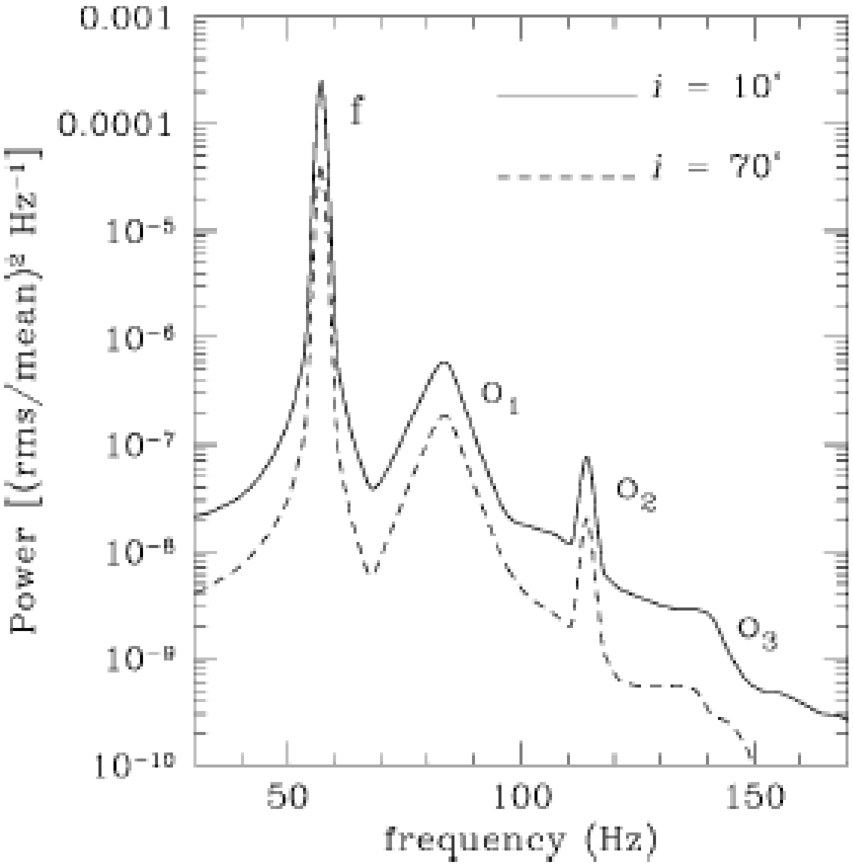
Resonance condition: $\omega_0 = (n/2)\,\omega_1$, $n = 1,2,3\ldots$

Suppose $\omega_1 < \omega_0$ (as true for epicyclic frequencies),

then first possibility $n=3 \Rightarrow$ 3:2 ratio.



Numerical GR hydro simulation of thick disks (accreting fat tori).
Ray tracing.



4. DISCUSSION AND CONCLUSIONS

We have demonstrated a positive correlation between the intrinsic normal-mode oscillations of a pressure-supported torus and the extrinsic X-ray light curves and power spectra as seen by a distant observer. In addition to this being the first ray-tracing calculation exploiting dynamically the results of relativistic hydrodynamics simulations, our investigation confirms the feasibility of the oscillating-torus model as an explanation for the integer ratios seen in high-frequency QPO peaks. The specific parameters of the torus model still require further investigation in order to best fit the QPO data, including a more comprehensive study of black hole mass, spin, and inclination angles.

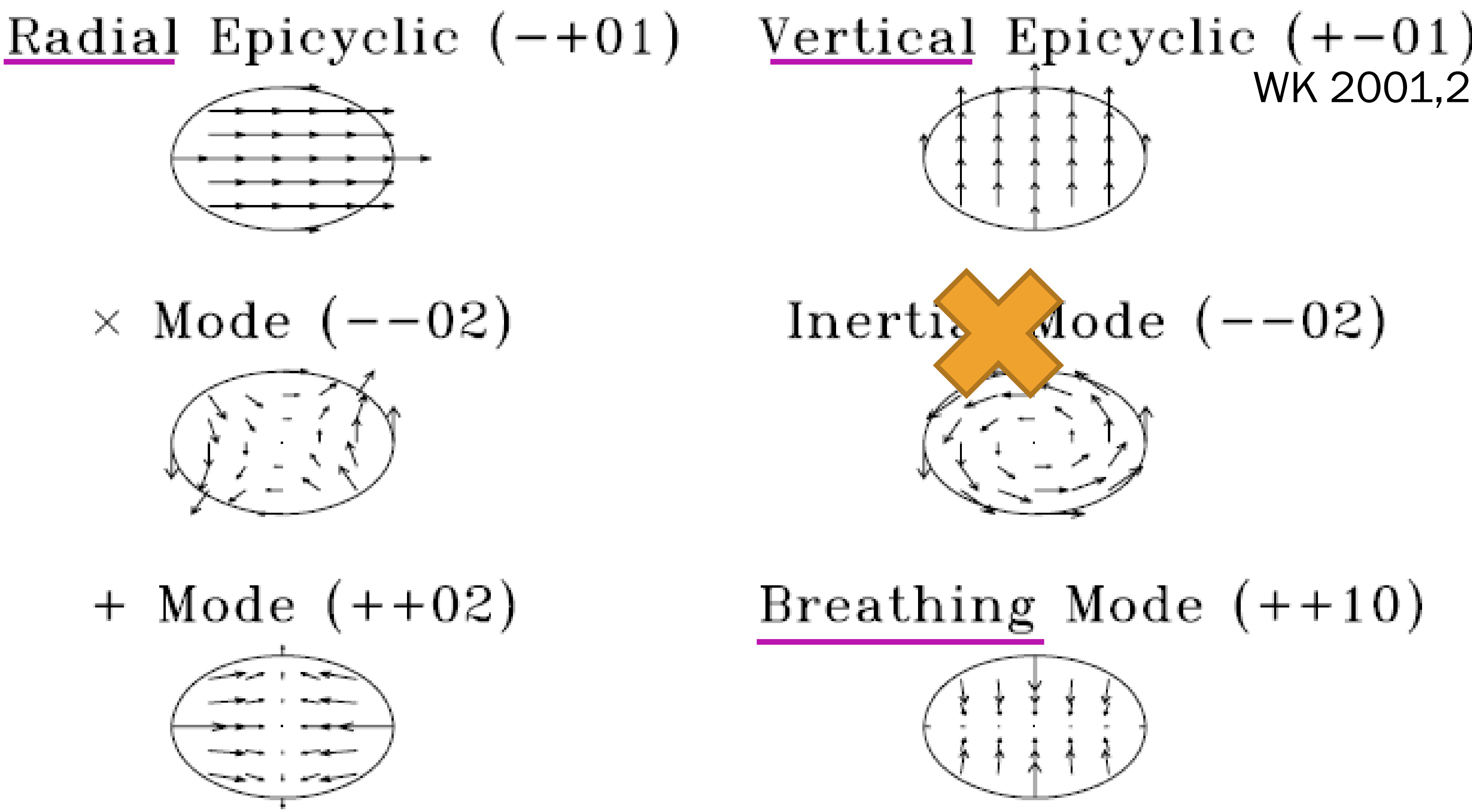
For the line emission models considered, the variation in the light curve is caused largely by the gravitational redshift of photons coming from different radii as the torus moves in and out of the black hole's potential well. Unlike the relativistic hot-spot model, for the same emission mechanism the oscillating-torus model predicts higher amplitude variations in the light curve for smaller inclination angles, while at higher angles the special relativistic beaming and gravitational lensing counter the gravitational redshift, reducing the variations in flux. On the other hand, the thermal emission model predicts

SOME QUESTIONS

- What is the relation of Rezzolla and collaborators' modes (2006, ...) to tori modes?
- Relation to disk oscillations?
- Is there evidence of resonance in our simulated tori or disks?
- If so, what is the nature of the resonance?

NB: Y. Kato (2004 PASP 56, 213) reported "first evidence for the excitation of resonant disk oscillations in MHD accretion flow." (pseudo-Newtonian study)

Oscillation modes of Keplerian **tori** well studied.
5 **modes** identified in our GR simulations:



WK 2001,2004

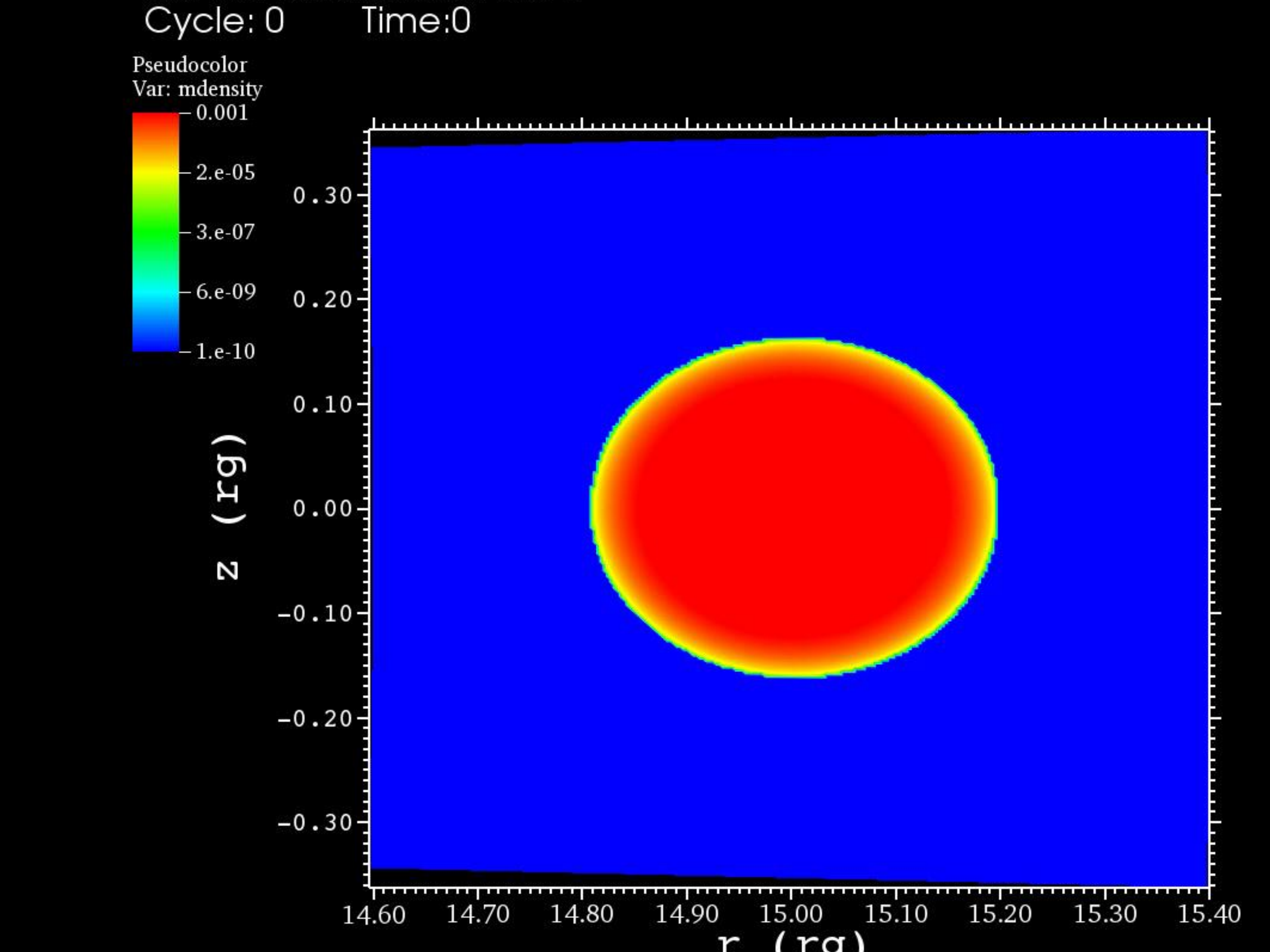
Blaes et al 2006

Think of a thin accretion disk as composed of tori =>
expect at least radial and vertical *local* oscillations.

THICK DISK/TORI OSCILLATIONS

Mishra, Vincent et al 2017: GRHD simulations with ray tracing for tori of all thicknesses:

- Perturbation excites several modes and their harmonics
- Reproduce thick disk simulations of Rezzolla and collaborators (2006, ...)
- => Identify modes of thick accreting torus with modes of slender tori


Cycle: 0 Time:0
Pseudocolor
Var: mdensity
0.001
2.e-05
3.e-07
6.e-09
1.e-10
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
z (rg)
14.60
14.70
14.80
14.90
15.00
15.10
15.20
15.30
15.40
r (rg)

B. Mishra et al.
MNRAS **467,** 4036–4049 (2017)

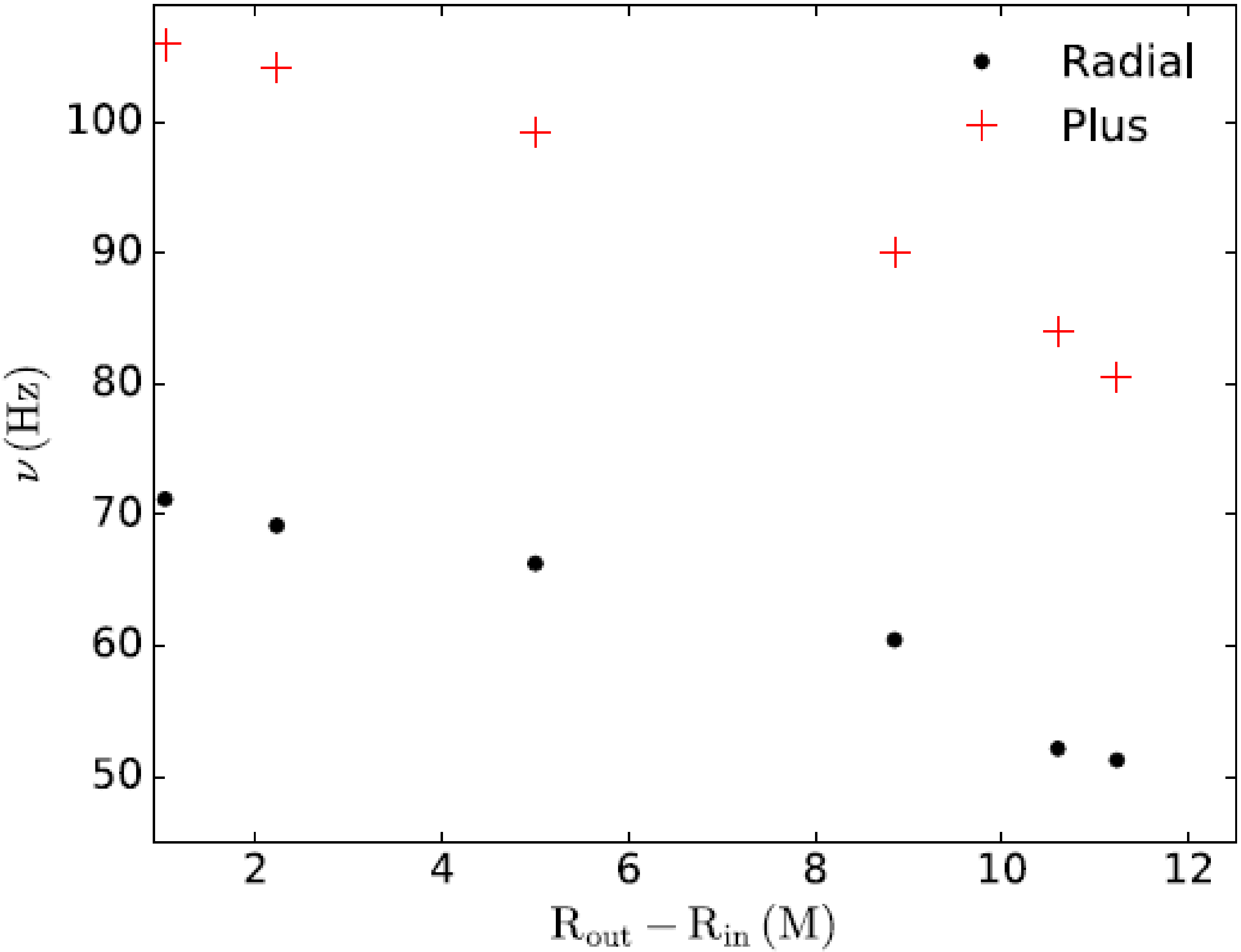
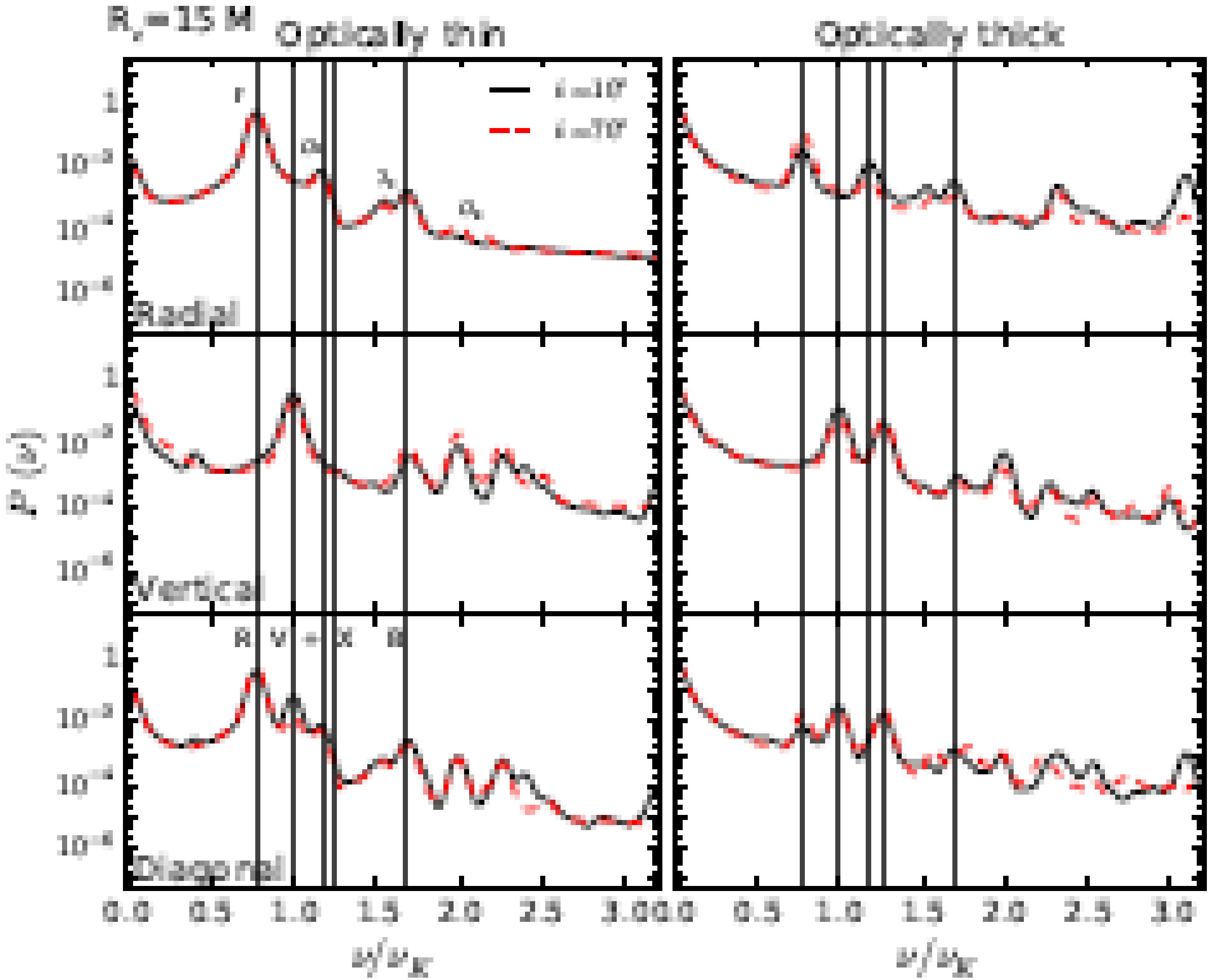


Figure C3. Radial (black, dots) and plus (red, plus) mode frequency plotted against the cross-section size of the torus. The torus pressure maximum is initially at $R_c = 8.352$, the polytropic index is $\gamma = 4/3$. The largest torus corresponds to model c of Zanotti et al. (2003).

vertical lines: Radial, Vertical, +, X, Breathing

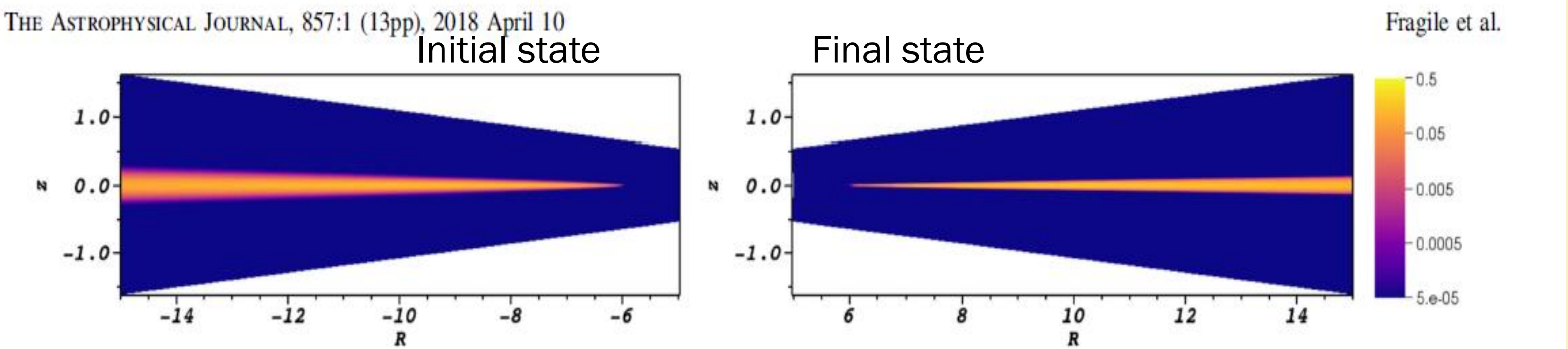
Mishra, Vincent ++ WK 2017

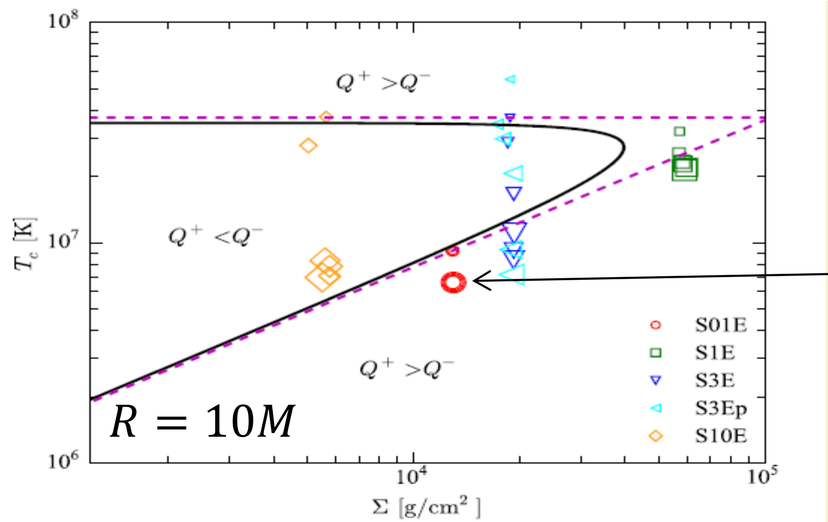


PDS at ∞
(ray traced)

GLOBAL THIN-DISK GR HYDRO SIMULATIONS

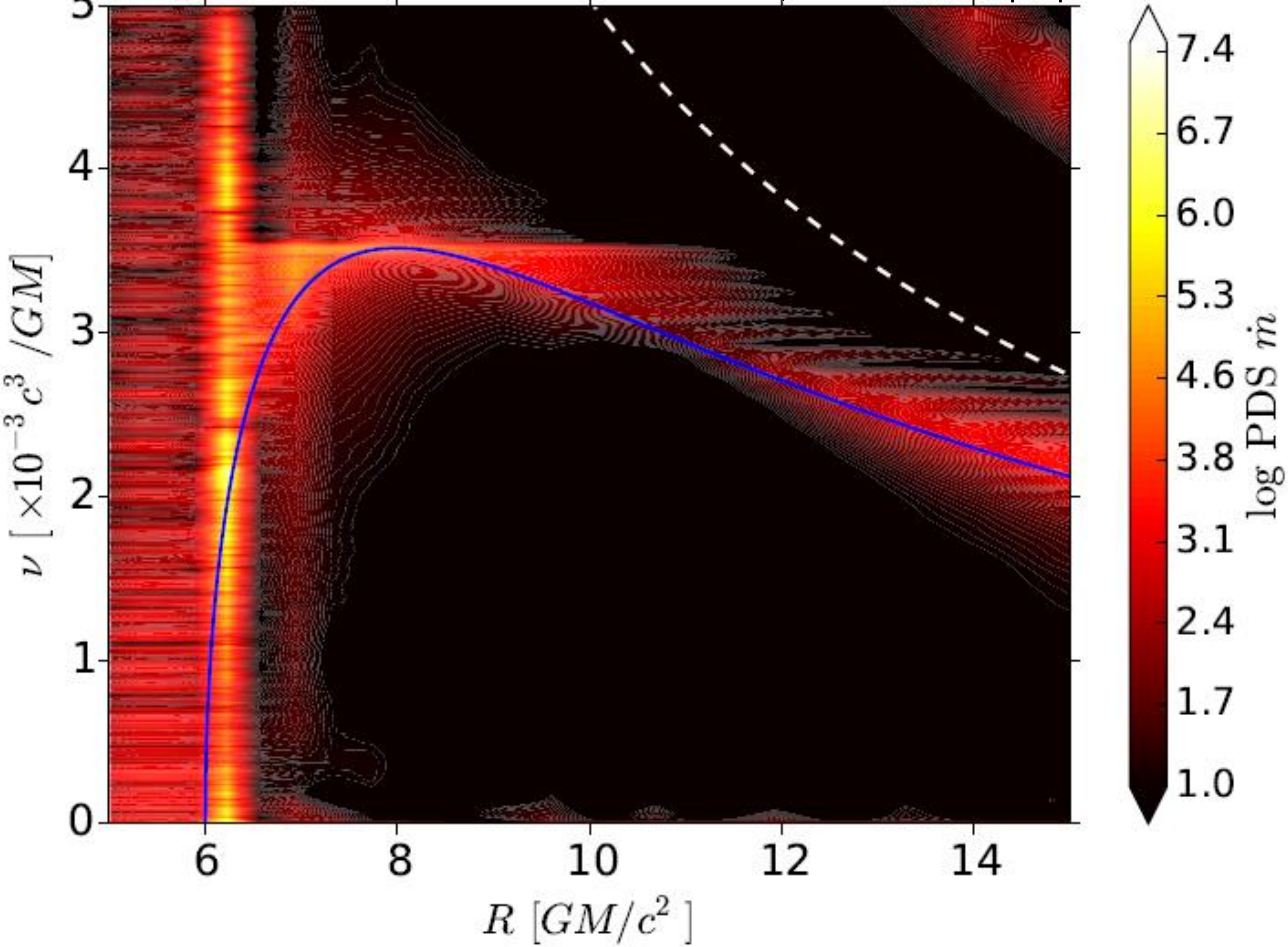
- Fragile, Etheridge, **Mishra**, WK, Anninos 2017
- Inevitable initial adjustment excites oscillation

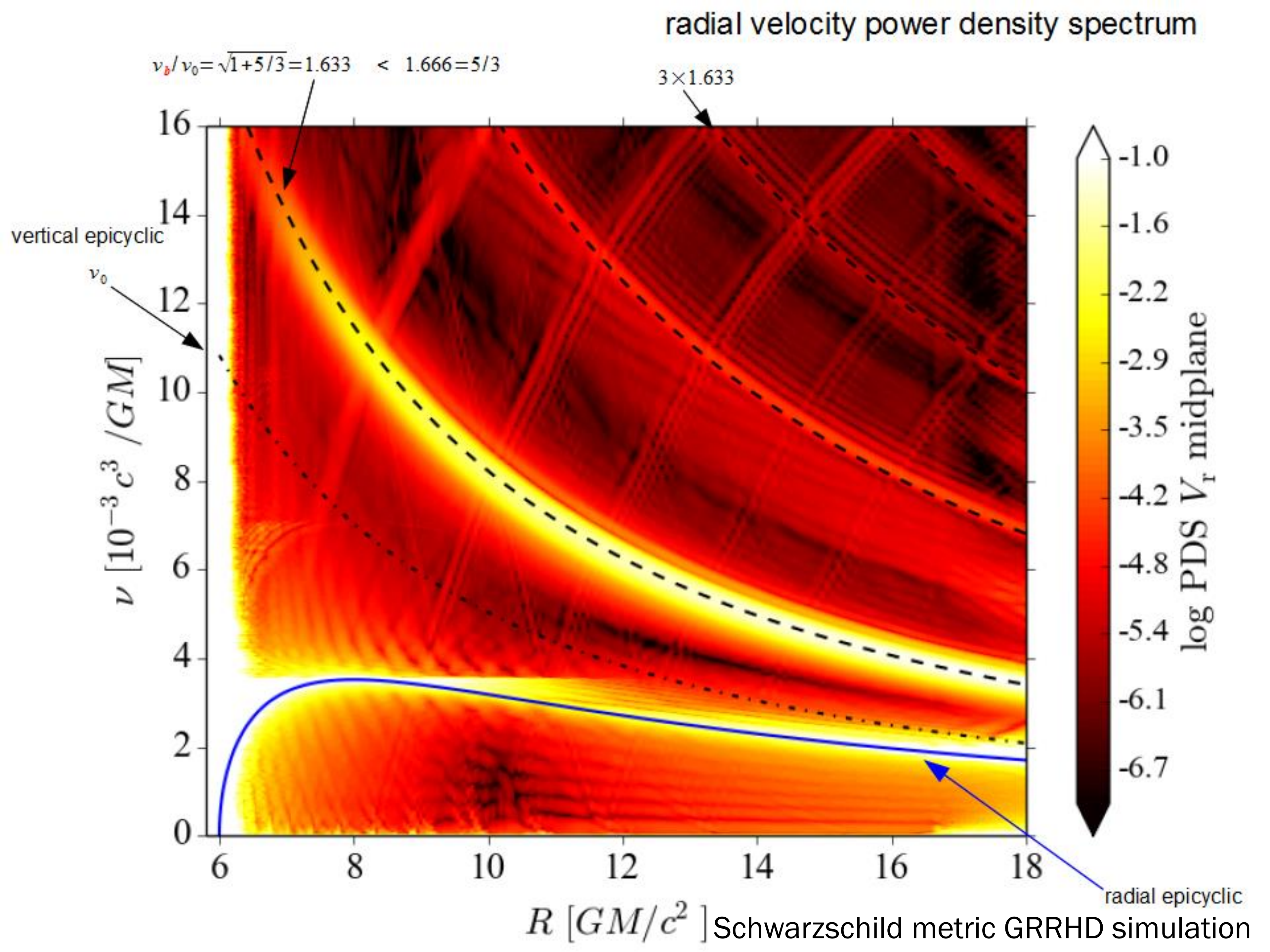




Reporting on this stable model

Mishra, WK + 2018 in prep





radial velocity power density spectrum
$\nu_b/\nu_0=\sqrt{1+5/3}=1.633 \quad < \quad 1.666=5/3$
3×1.633
vertical epicyclic
ν_0
$\nu\ [10^{-3}c^3\ /GM]$
$R\ [GM/c^2]$
log PDS V_r midplane
radial epicyclic
Schwarzschild metric GRRHD simulation

THIN-DISK PLANE-PARALLEL APPROXIMATION

Breathing mode $\omega_2 = \sqrt{1+\Gamma}$:

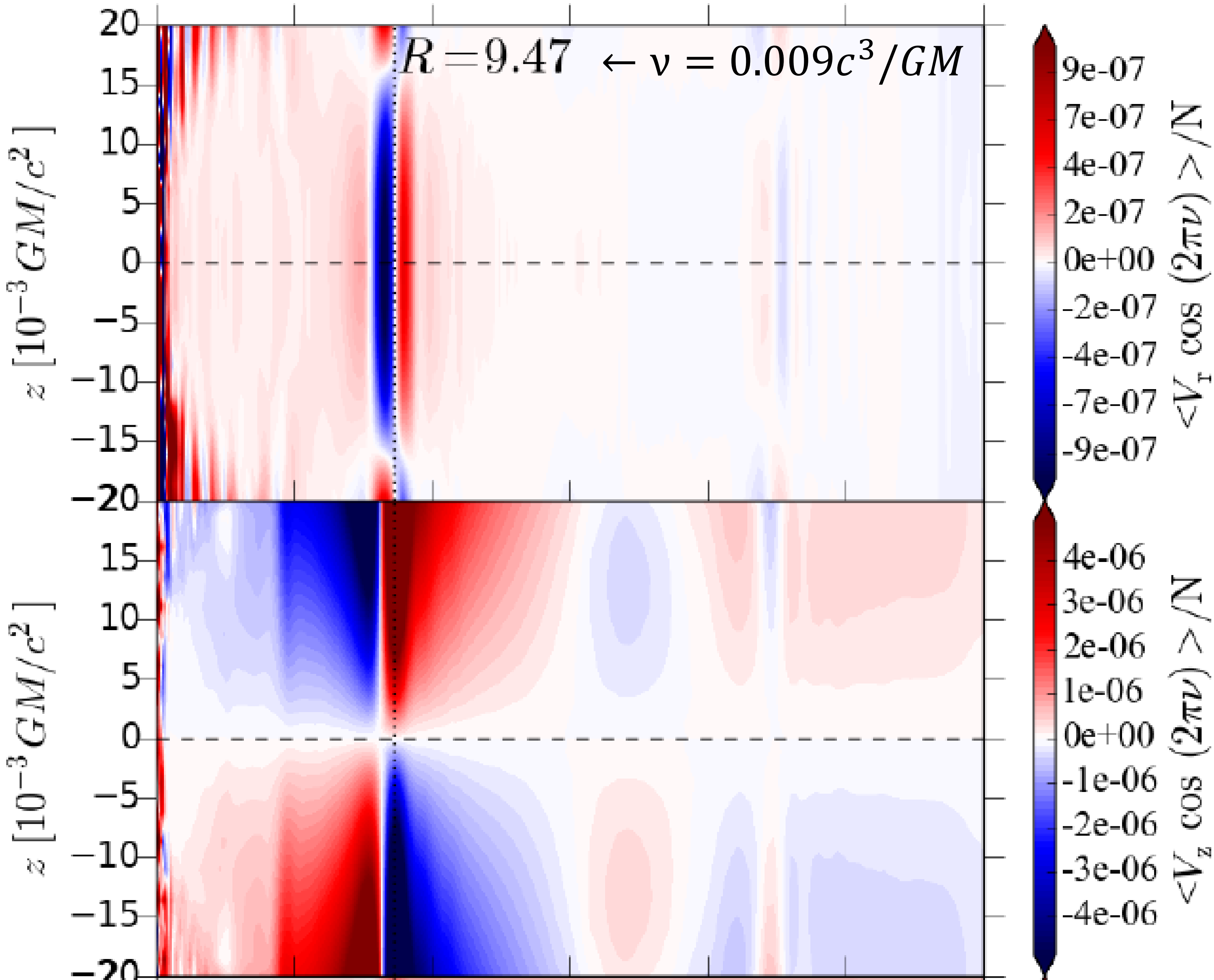
- Isothermal oscillations,
  Lubow & Pringle 1993, Kato 2001.
- Adiabatic oscillations (adiabatic index $\Gamma = 1 + 1/n$):
  Silbergleit, Wagoner & Ortega-Rodriguez 2001, Bollimpalli & WK 2017:

Gegenbauer polynomial solutions, $m = 0,1,2,\ldots$

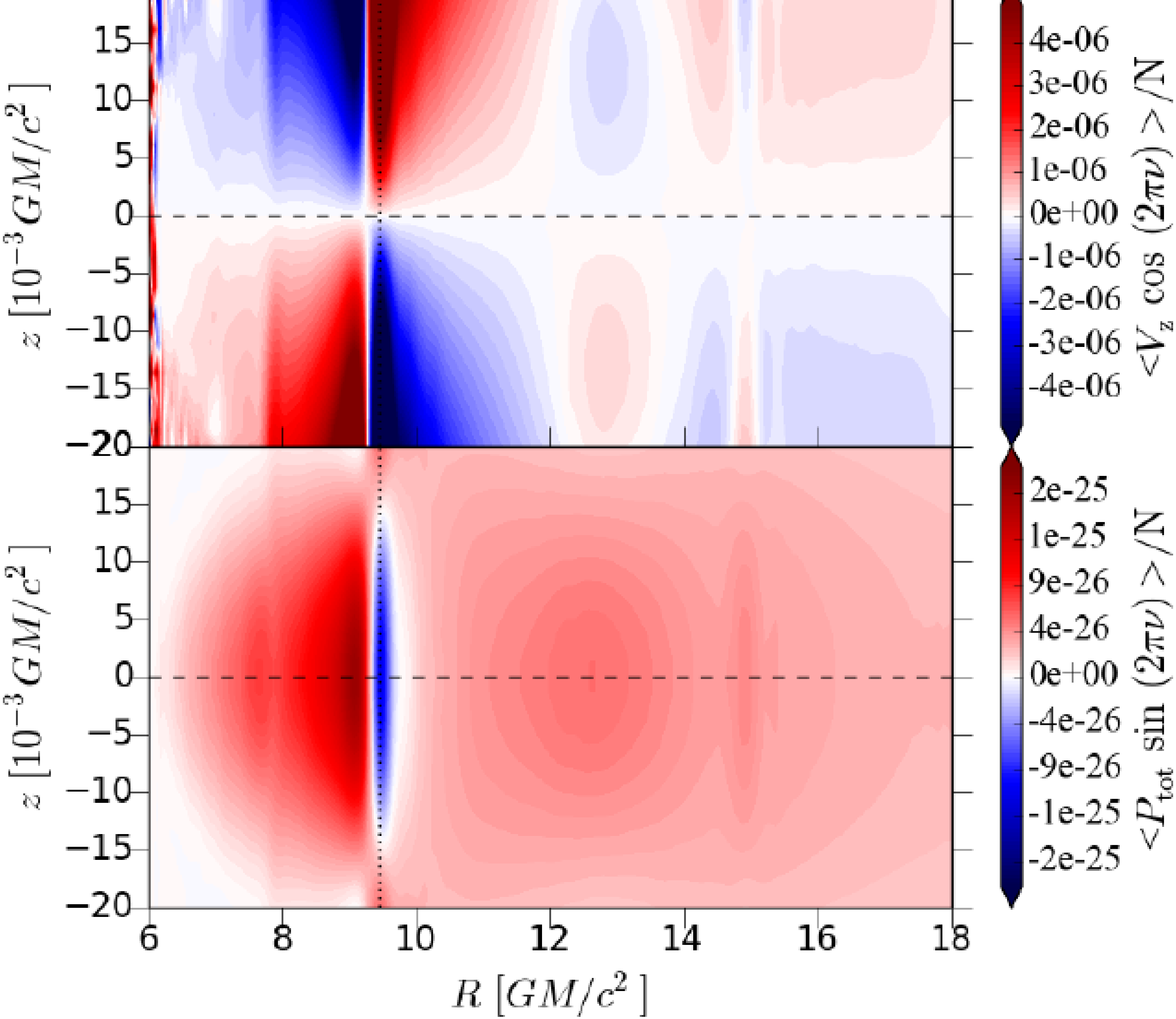
frequency $\omega_m = \sqrt{\frac{m(m+2n-1)}{2n}}$

(in units of vertical epicyclic frequency ω_1)

with breathing mode for $m = 2$



$R = 9.47$ $\leftarrow \nu = 0.009c^3/GM$
$z\ [10^{-3}GM/c^2]$
$<V_r \cos(2\pi\nu)>/N$
$<V_z \cos(2\pi\nu)>/N$
20
15
10
5
0
−5
−10
−15
−20
9e-07
7e-07
4e-07
2e-07
0e+00
-2e-07
-4e-07
-7e-07
-9e-07
4e-06
3e-06
2e-06
1e-06
0e+00
-1e-06
-2e-06
-3e-06
-4e-06



15
10
5
0
−5
−10
−15
−20
$z\ [10^{-3}GM/c^2]$
4e-06
3e-06
2e-06
1e-06
0e+00
-1e-06
-2e-06
-3e-06
-4e-06
$<V_z \cos(2\pi\nu)>/N$
2e-25
1e-25
9e-26
4e-26
0e+00
-4e-26
-9e-26
-1e-25
-2e-25
$<P_{tot} \sin(2\pi\nu)>/N$
6
8
10
12
14
16
18
$R\ [GM/c^2]$

(associated with pressure variations)

Breathing mode in an accretion disk

TRANSIENT BREATHING LOCAL OSCILLATIONS

- Observed in global thin disk GRRHD simulations
- "Eigen-" function determined numerically
- Radial breathing motion accompanies the vertical one
- Vertical + breathing oscillations may explain the observed 69 Hz & 42 Hz black hole QPOs in GRO 1915+20.

SUMMARY

- Oscillations of tori (all sizes) well understood.
- Global modes and local oscillations present in GRHydro disks.
- Some evidence for resonance in simulated accretion disks.