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Note

Numerical Models of Slim Accretion Disks

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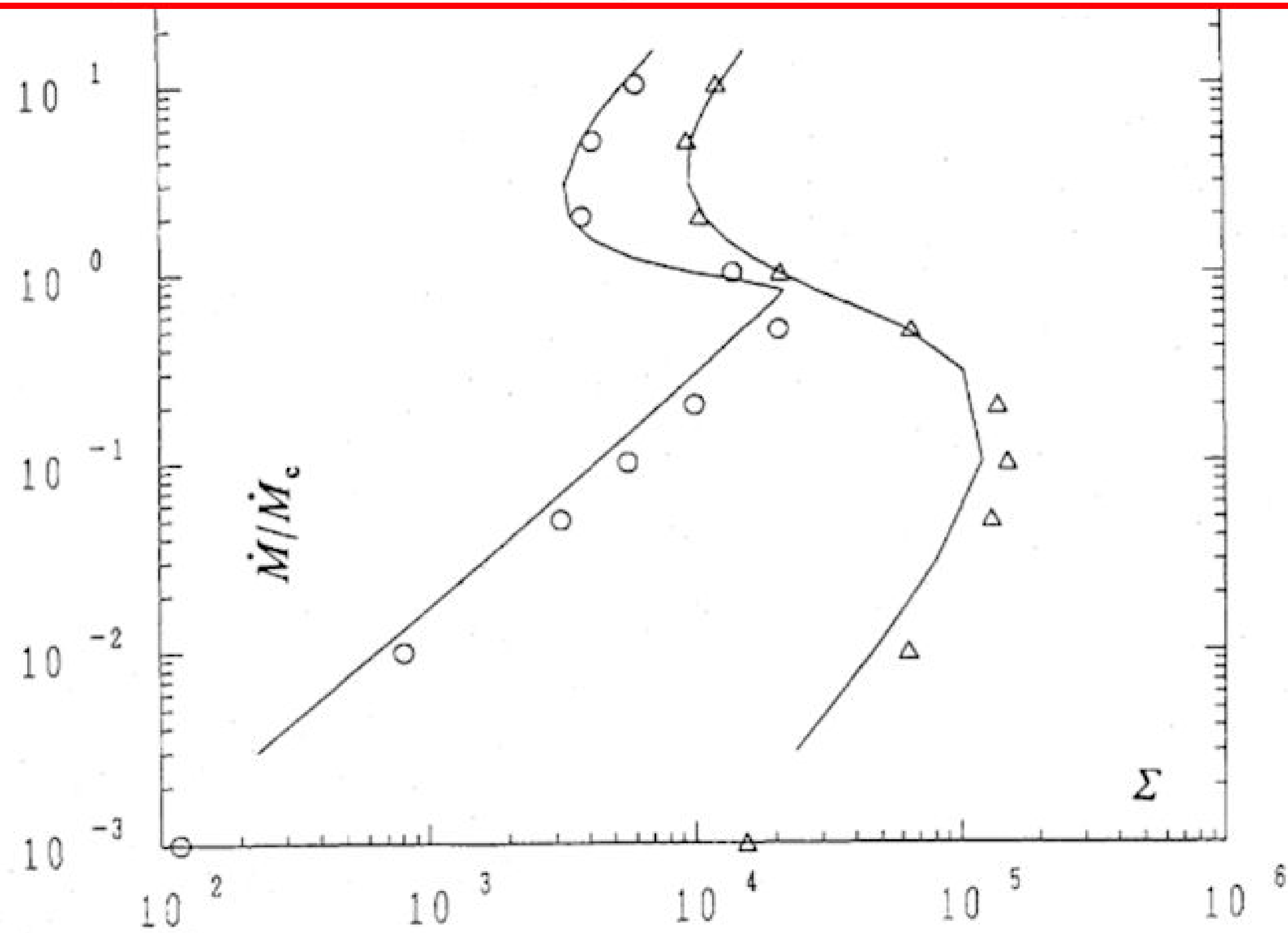
Fig. 1. Kyoto (solid lines) and Trieste (circles and triangles) models of slim accretion disks for two particular radii ($r=3r_g$ (left) and $r=4r_g$ (right)) and for the mass of the central object $M=10M_\odot$ and Shakura–Sunyaev (1973) viscosity parameter $\alpha=10^{-2}$.



Fundamental limits for power: Planck and Eddington

$$\frac{c^5}{G}$$

The Planck power (i.e. power expressed in Planck's units) equals

$$L_{\rm Planck} = \frac{c^5}{G} \approx 10^{58}\ [\rm erg\, sec^{-1}] = 10^{52}\ [\rm Watts].$$

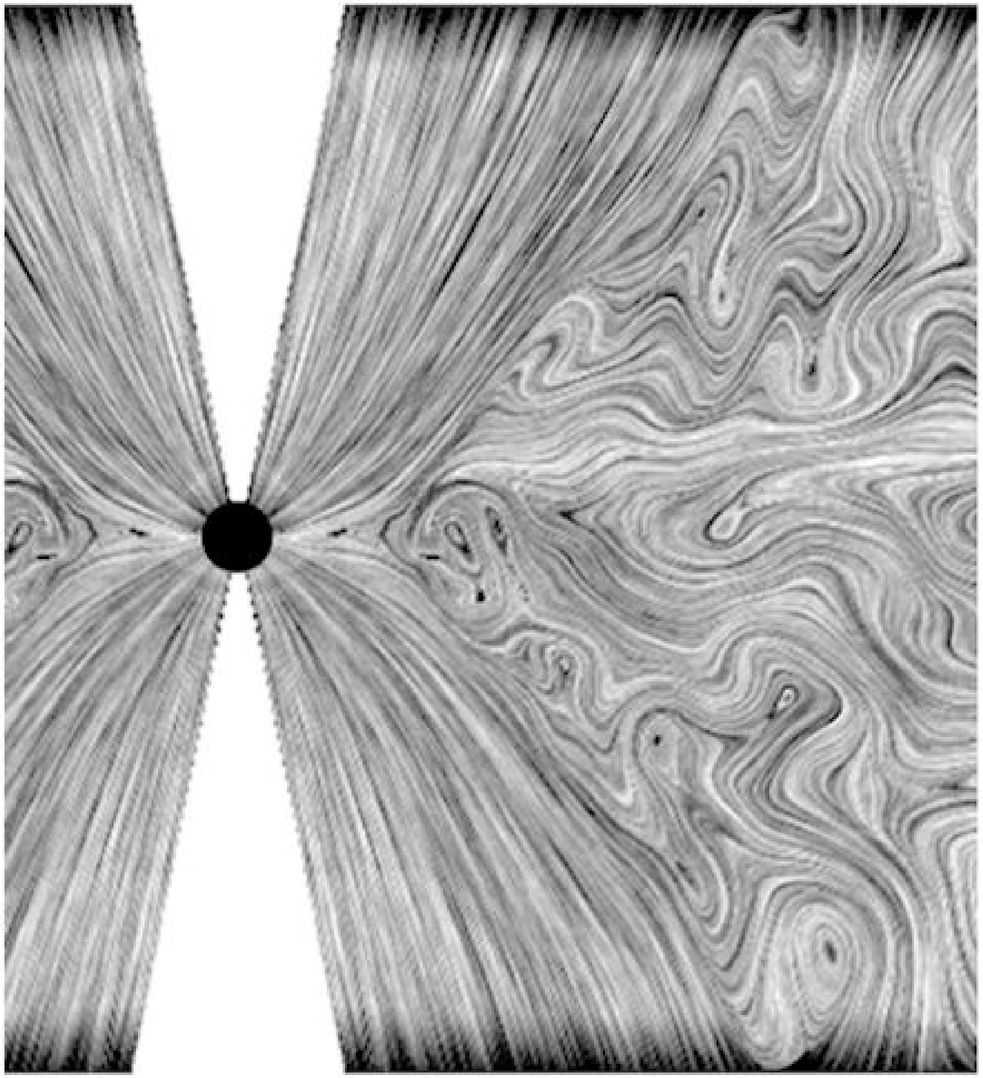
Rather surprisingly, it does not depend on the Planck constant h. The maximal energy available from an object with the mass M (and gravitational radius $R_{\rm G} = GM/c^2$) is $E_{\max} = Mc^2$. The minimal time in which this energy may be liberated is $t_{\min} = R_{\rm G}/c$. Thus, the maximal power $L_{\max} = E_{\max}/t_{\min} = c^5/G = L_{\rm Planck}$. This is the absolute upper limit for power of anything in the Universe: all objects, phenomena, explosions, and evil empires[6].

$$L_{\rm Edd} = L_{\rm Planck}\frac{\Sigma_{\rm grav}}{\Sigma_{\rm rad}} = \frac{4\pi G M m_{\rm P} c}{\sigma_{\rm T}} = 1.4 \times 10^{38}\left(\frac{M}{M_\odot}\right)[\rm erg\ sec^{-1}]$$

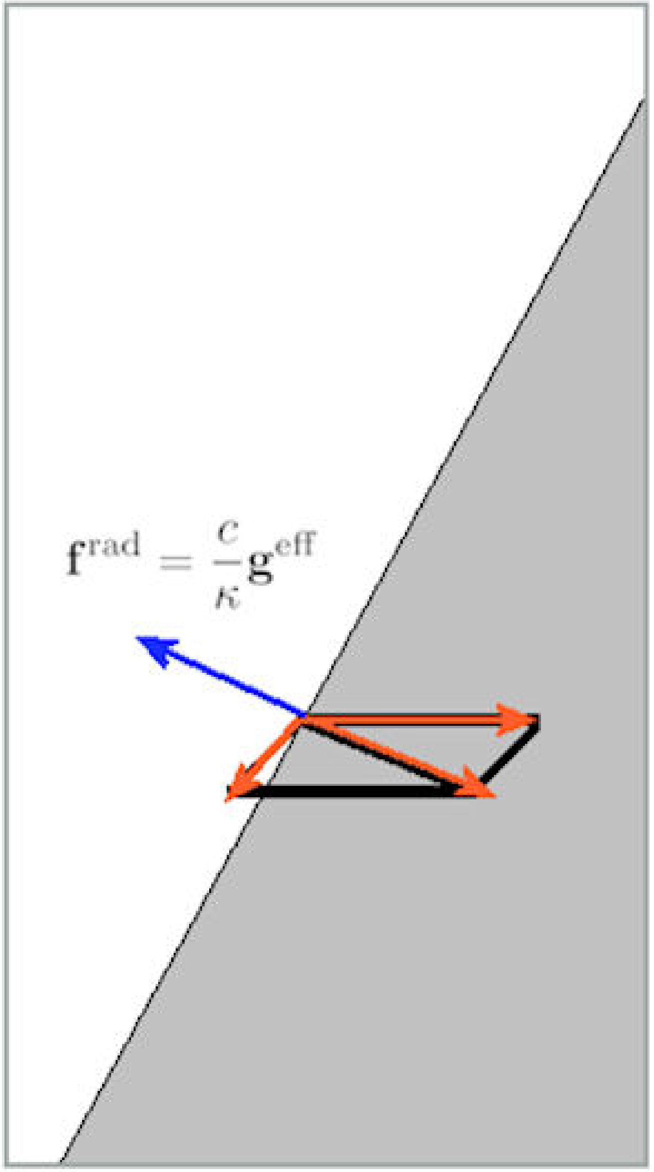
$$\Sigma_{\rm rad} = N\sigma_T \qquad N = M/m_{\rm P} \qquad \Sigma_{\rm grav} = 4\pi R_G^2$$

In no circumstances $L_{\rm Edd}$ can grow above $L_{\rm Planck}$. I noticed [43] that the two limits are equal when $M \approx N_0 m_{\rm P}$, with $N_0 = N^2_{\rm Dirac}/3$, where the Dirac number $N_{\rm Dirac} = e^2/Gm_{\rm e}m_{\rm P}$ equals the ratio of Coulomb's to Newton's force between electron and proton. Then $N_0 \approx N_{\rm Edd} \equiv 136 \times 2^{256} \approx 1.6 \times 10^{79}$, where $N_{\rm Edd}$ is the Eddington number that played an important role in Eddington's *Fundamental Theory* [44]. The number was introduced by Eddington's immortal statement, *I believe there are 15 747 724 136 275 002 577 605 653 961 181 555 468 044 717 914 527 116 709 366 231 425 076 185 631 031 296 protons in the universe and the same number of electrons.* One thus may write, $\boxed{L_{\rm Edd} = (N/N_{\rm Edd})\, L_{\rm Planck}.}$ A very Eddingtonish connection indeed, embracing his luminosity and his number, of which he was not aware.

An idea on the black hole accretion



The photosphere shape: equations



$$\frac{1}{\rho}\frac{\partial p}{\partial R} = \frac{\partial \Phi}{\partial R} + \frac{\ell^2(R,Z)}{R^3}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial Z} = \frac{\partial \Phi}{\partial Z}$$

$$p(R,Z) = 0 \;\Downarrow\; Z = H(R)$$

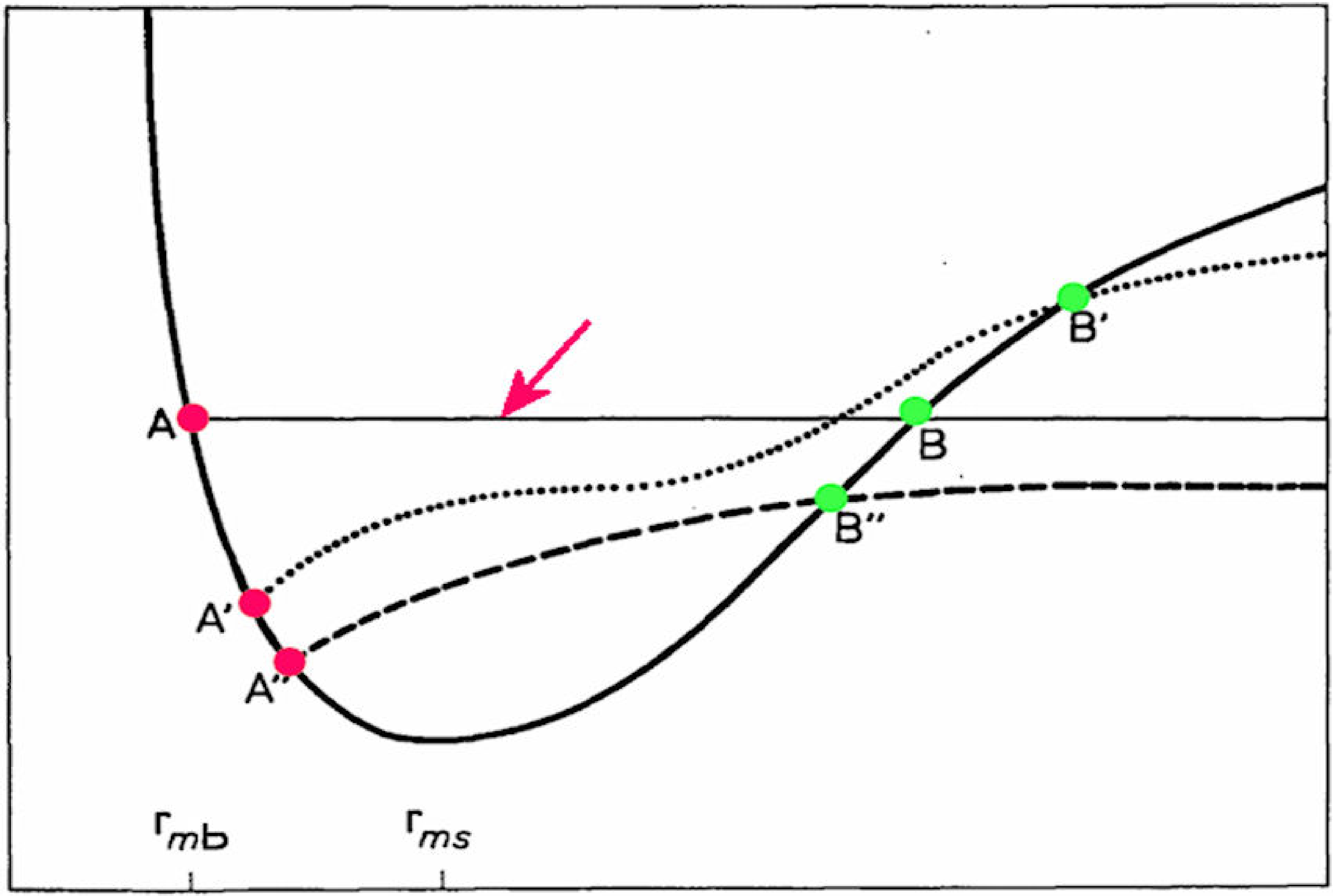
$$\frac{dH}{dR} = -\left\{ \frac{\frac{\partial \Phi}{\partial Z}}{\frac{\partial \Phi}{\partial R} + \frac{\ell^2(R)}{R^3}} \right\}_{Z=H}$$

$$\Downarrow$$

$$\ell^2(R,Z)_{Z=H} \quad \ell^2(R) \Rightarrow H = H(R)$$

$$\Phi = -\frac{GM}{r - r_{\rm G}}, \quad r_{\rm G} = \frac{2GM}{c^2}$$

$$r = \left(R^2 + Z^2\right)^{1/2}$$


A
A'
A''
B
B'
B''
r_{mb}
r_{ms}

Thin, Keplerian

$$\frac{1}{\rho}\frac{\partial p}{\partial Z} = -\frac{\partial \Phi}{\partial Z} \quad \Rightarrow \quad C_S^2 = V_K^2 \left(\frac{H}{R}\right)^2$$

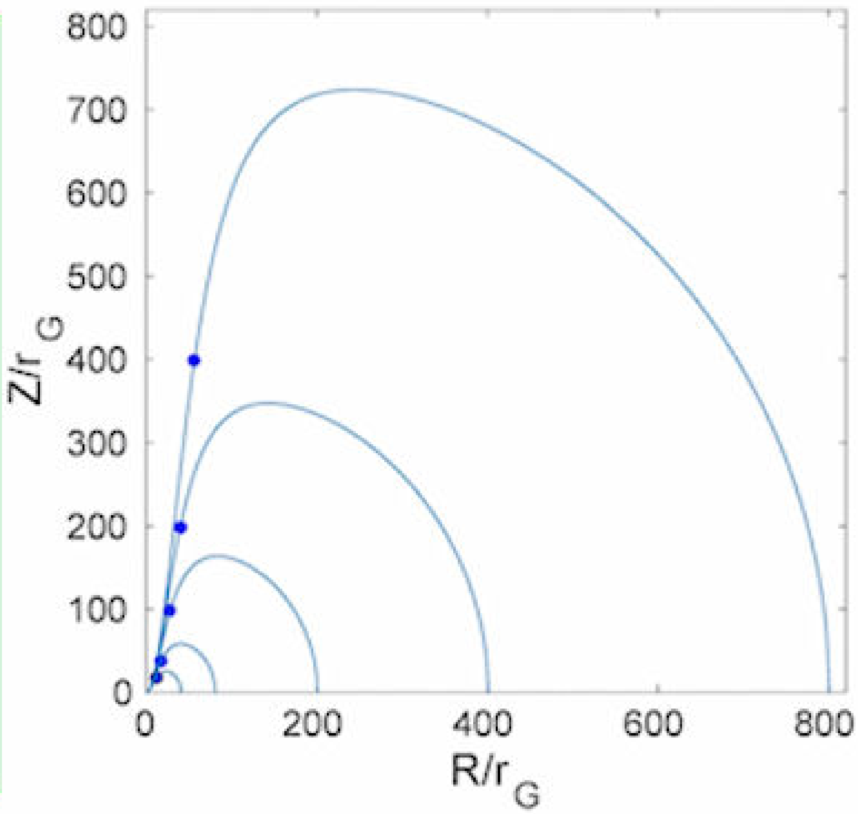
$$\frac{1}{\rho}\frac{\partial p}{\partial R} \sim \frac{C_S^2}{R} \sim \frac{V_K^2}{R}\left(\frac{H}{R}\right)^2 \sim \frac{1}{R^3}\left(\ell_K^2 - \ell^2\right)$$

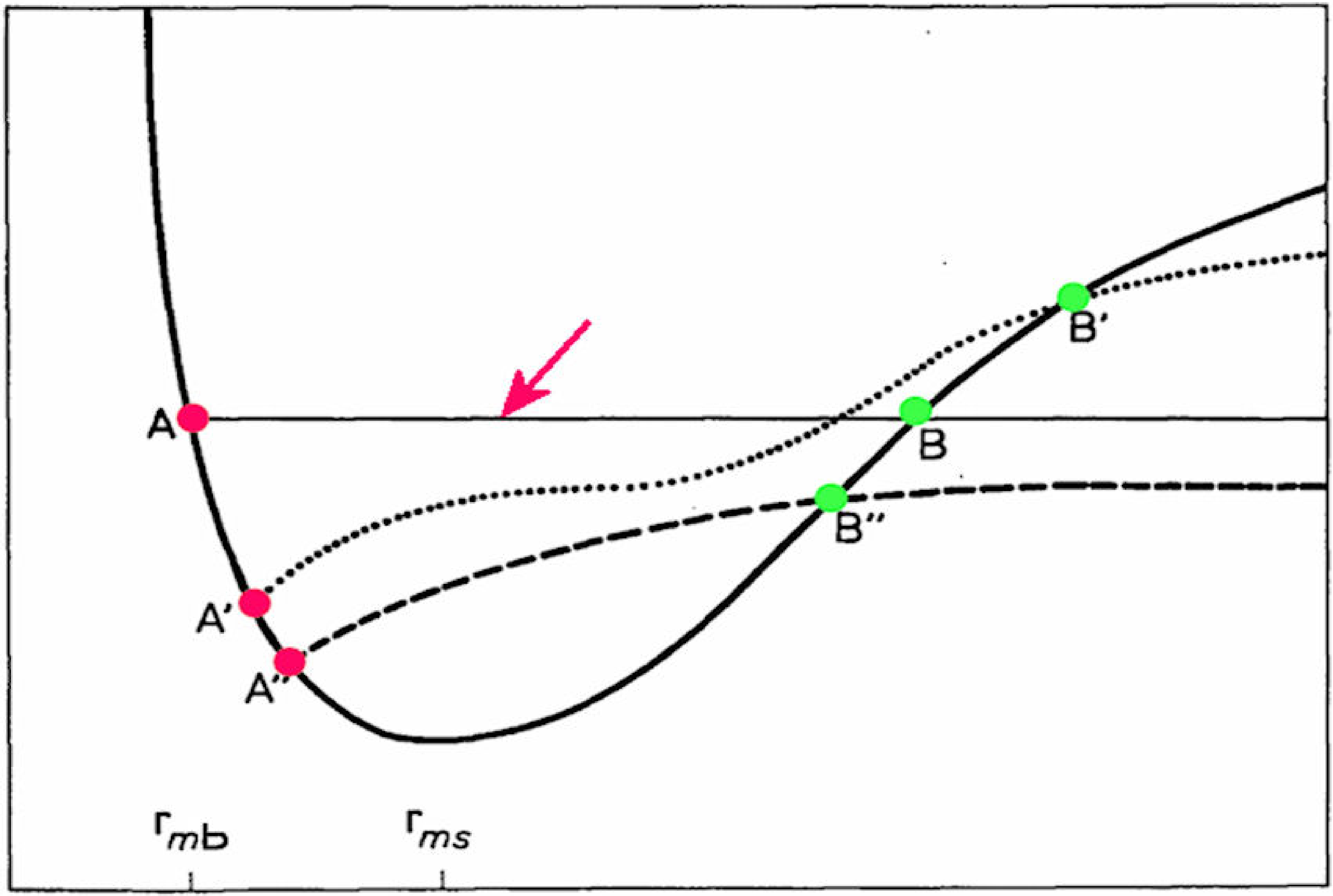
$$\ell_K^2 - \ell^2 \sim \left(\frac{H}{R}\right)^2 \ll 1 \quad \Rightarrow \quad \ell = \ell_K$$

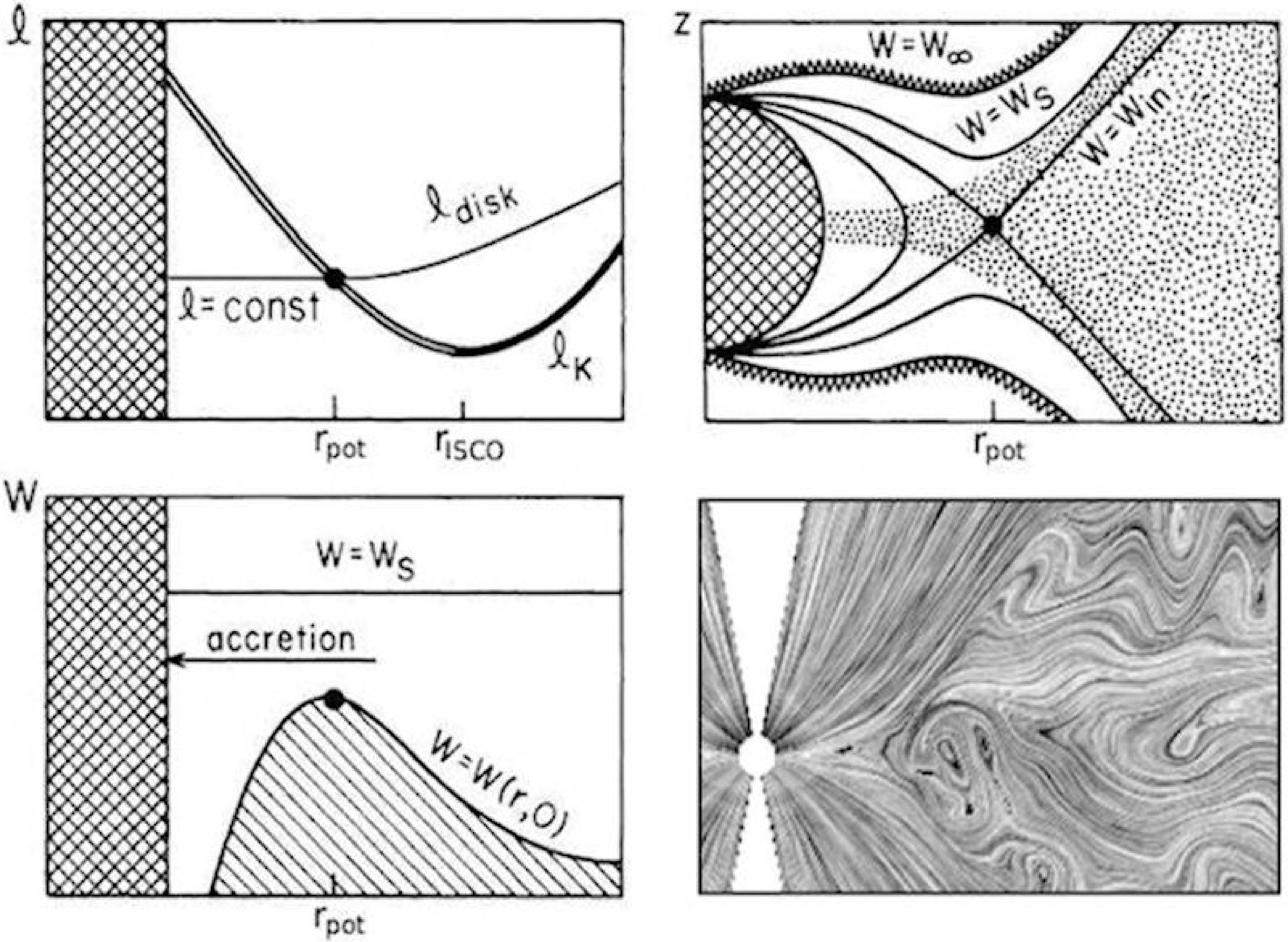
$$\frac{dH}{dR} = - \left\{ \frac{\frac{\partial \Phi}{\partial Z}}{\frac{\partial \Phi}{\partial R} + \frac{\ell^2}{R^3}} \right\}_{Z=H}$$

$$\Downarrow$$

$$H = H(R)$$




A
A'
A''
B
B'
B''
r_{mb}
r_{ms}



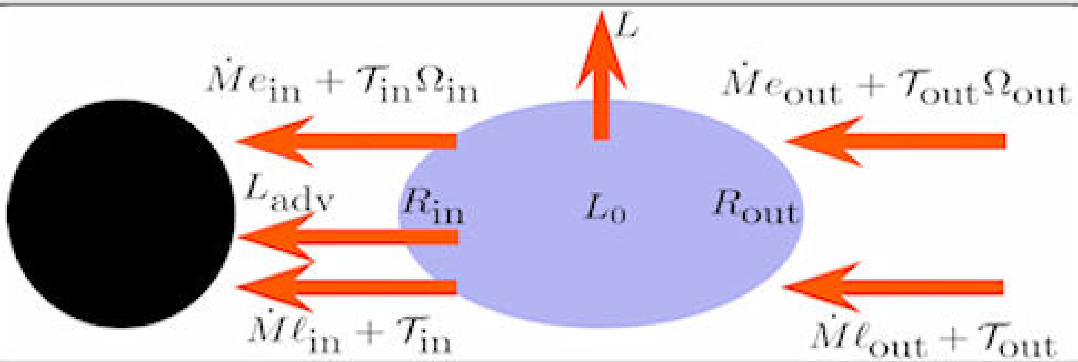
ℓ
ℓ disk
ℓ= const
ℓ K
r pot
r ISCO
z
W = W∞
W = W S
W = W in
r pot
W
W = W S
accretion
W = W(r,0)
r pot

Polish Doughnuts: the luminosity

$$\begin{aligned} L &= \int \mathbf{f}^{\mathrm{rad}} d\mathbf{S} \\ &= \frac{4\pi c}{\kappa} \int\limits_{R_{\mathrm{in}}}^{R_{\mathrm{out}}} \left(\frac{\partial \Phi}{\partial Z}\right)_{Z=H} \left[1 + \left(\frac{dH}{dR}\right)^2\right] R\, dR. \end{aligned}$$

$$H = H(R)$$

The efficiency of accretion



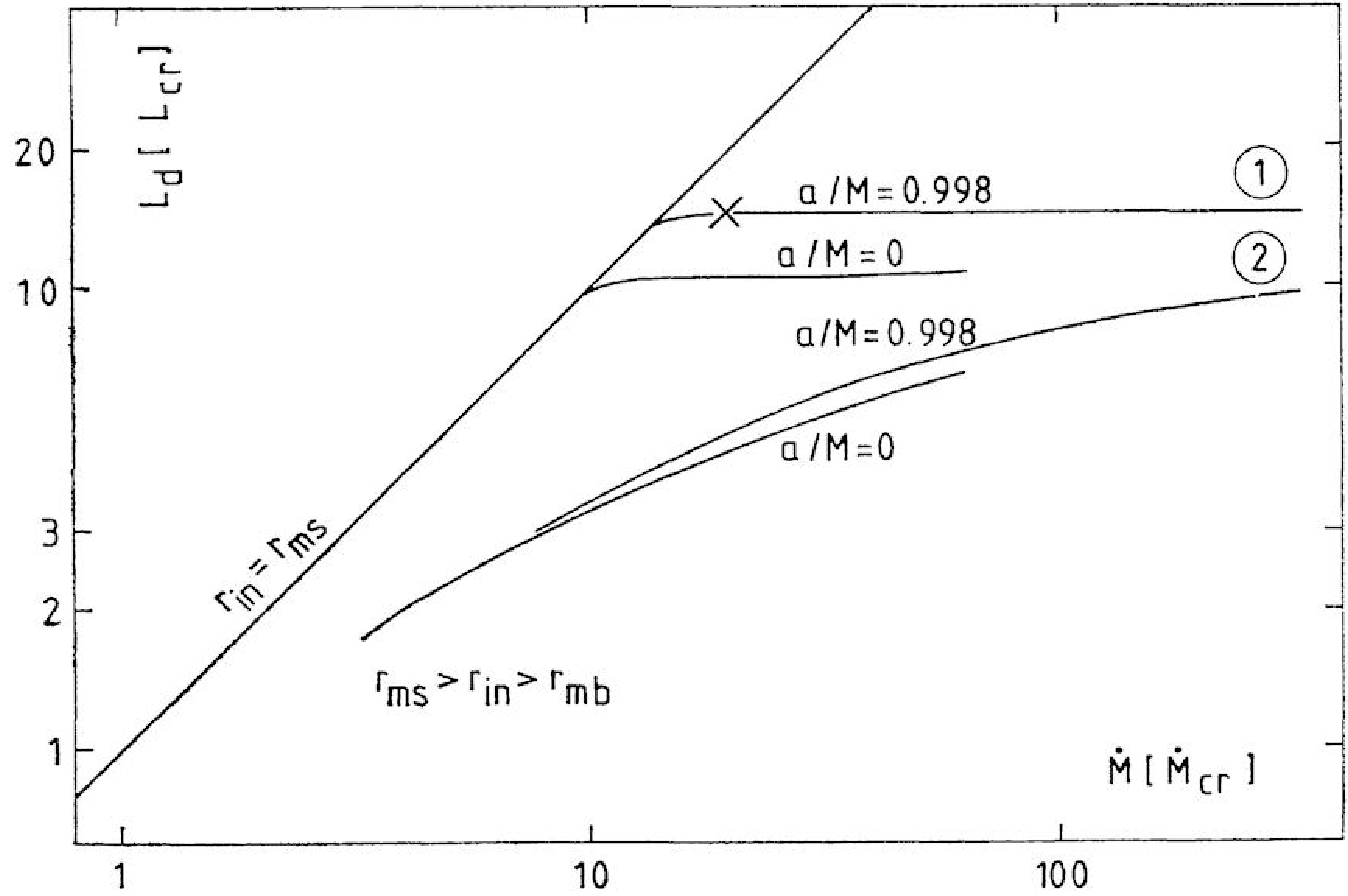
$$(\dot{M}e_{\rm out} + \mathcal{T}_{\rm out}\Omega_{\rm out}) - (\dot{M}e_{\rm in} + \mathcal{T}_{\rm in}\Omega_{\rm in}) = L_{\rm o},$$
$$(\dot{M}\ell_{\rm out} + \mathcal{T}_{\rm out}) - (\dot{M}\ell_{\rm in} + \mathcal{T}_{\rm in}) = 0.$$

$$e = \Phi + \frac{\ell^2}{2R^2} \quad \mathcal{T}_{\rm in} = 0 \quad L_{\rm o} = -\epsilon(R_{\rm in}, R_{\rm out})\,\dot{M}$$

$$\epsilon(R_{\rm in}, R_{\rm out}) = (e_{\rm out} - e_{\rm in}) - \Omega_{\rm out}(\ell_{\rm out} - \ell_{\rm in}).$$

$$(\text{total heating}: \; L_{\rm o}) = (\text{total cooling}: \; L + L_{\rm adv})$$

$$L_{\rm adv} = \xi L_{\rm o} \qquad \boxed{\dot{M} = \frac{1}{\epsilon(1-\xi)} L}$$



$L_d\ [L_{cr}]$
20
10
3
2
1
1
10
100
$\dot{M}\ [\dot{M}_{cr}]$
$a/M = 0.998$
$a/M = 0$
$a/M = 0.998$
$a/M = 0$
1
2
$r_{in} = r_{ms}$
$r_{ms} > r_{in} > r_{mb}$

Polish Doughnuts: the idea

$$\ell = \ell(R) \text{ at } H(R)$$

$$\Downarrow$$

$$H = H(R),\ R_{\rm in} \Rightarrow \text{efficiency } \epsilon = \epsilon(R_{\rm in})$$

$$\Downarrow$$

$$\mathbf{g}_{\rm eff} = \mathbf{g}_{\rm eff}(R) \Rightarrow \mathbf{f}_{\rm rad}(R), \text{ and blackbody}$$

$$\Downarrow \qquad \Downarrow$$

spectrum

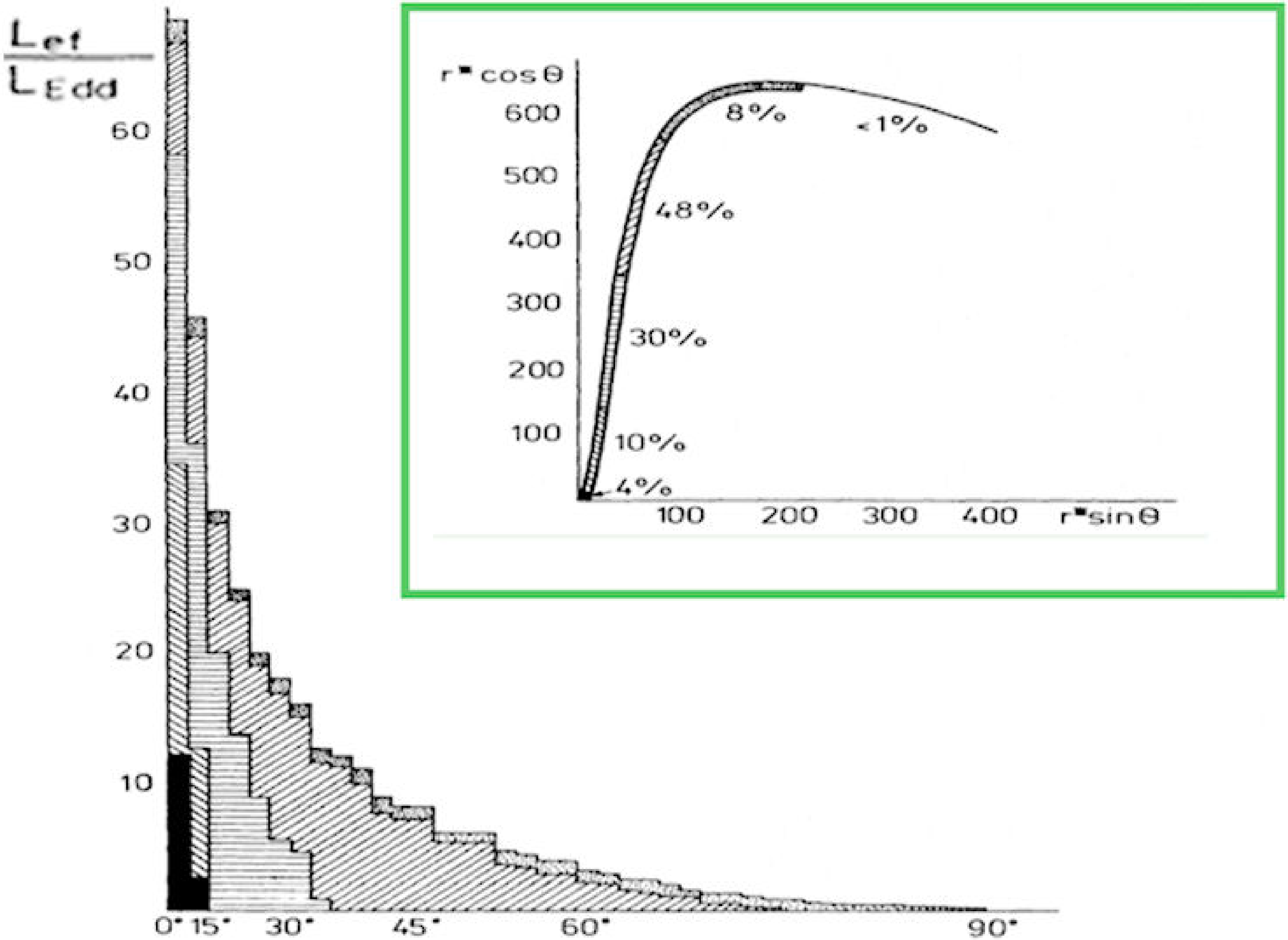
$$L = \epsilon \dot{M} c^2 \Leftarrow L = \int_{R_{\rm in}}^{\infty} \mathbf{f}_{\rm rad}(R)\, dA$$

$\mathbf{f}_{\rm rad}$ and inclination

$$\Downarrow$$

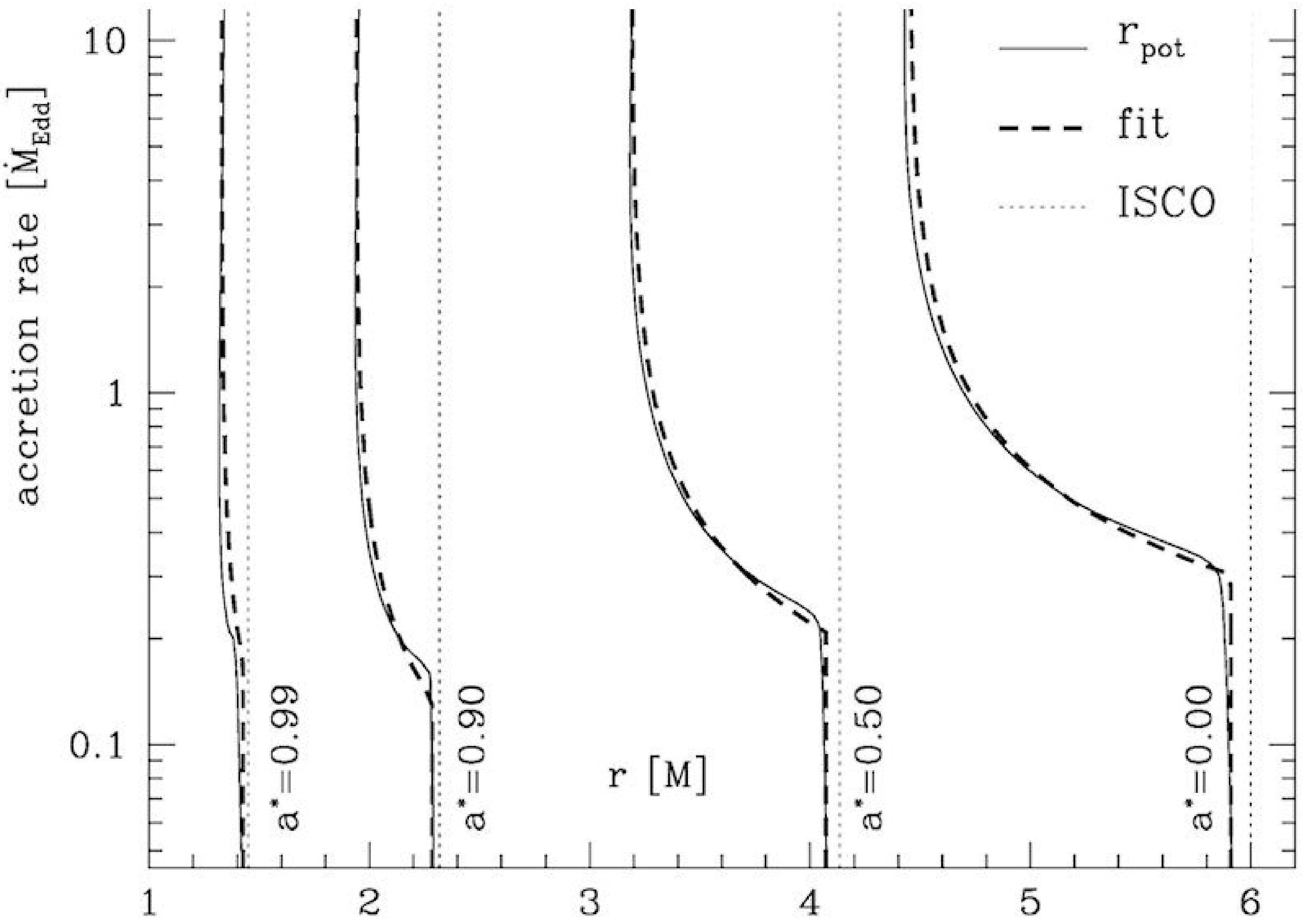
inclination dependent luminosity and spectrum

Marek Sikora: MNRAS 196, 257-268 (1981)



Leaving the innermost stable circular orbit: the inner edge of a black-hole accretion disk at various luminosities

M. A. Abramowicz[1,2,5,7], M. Jaroszyński[3], S. Kato[4], J.-P. Lasota[5,6], A. Różańska[2], and A. Sądowski[2]



accretion rate [$\dot{M}_{Edd}$]
10
1
0.1
r [M]
1
2
3
4
5
6
r_{pot}
fit
ISCO
$a^*=0.99$
$a^*=0.90$
$a^*=0.50$
$a^*=0.00$

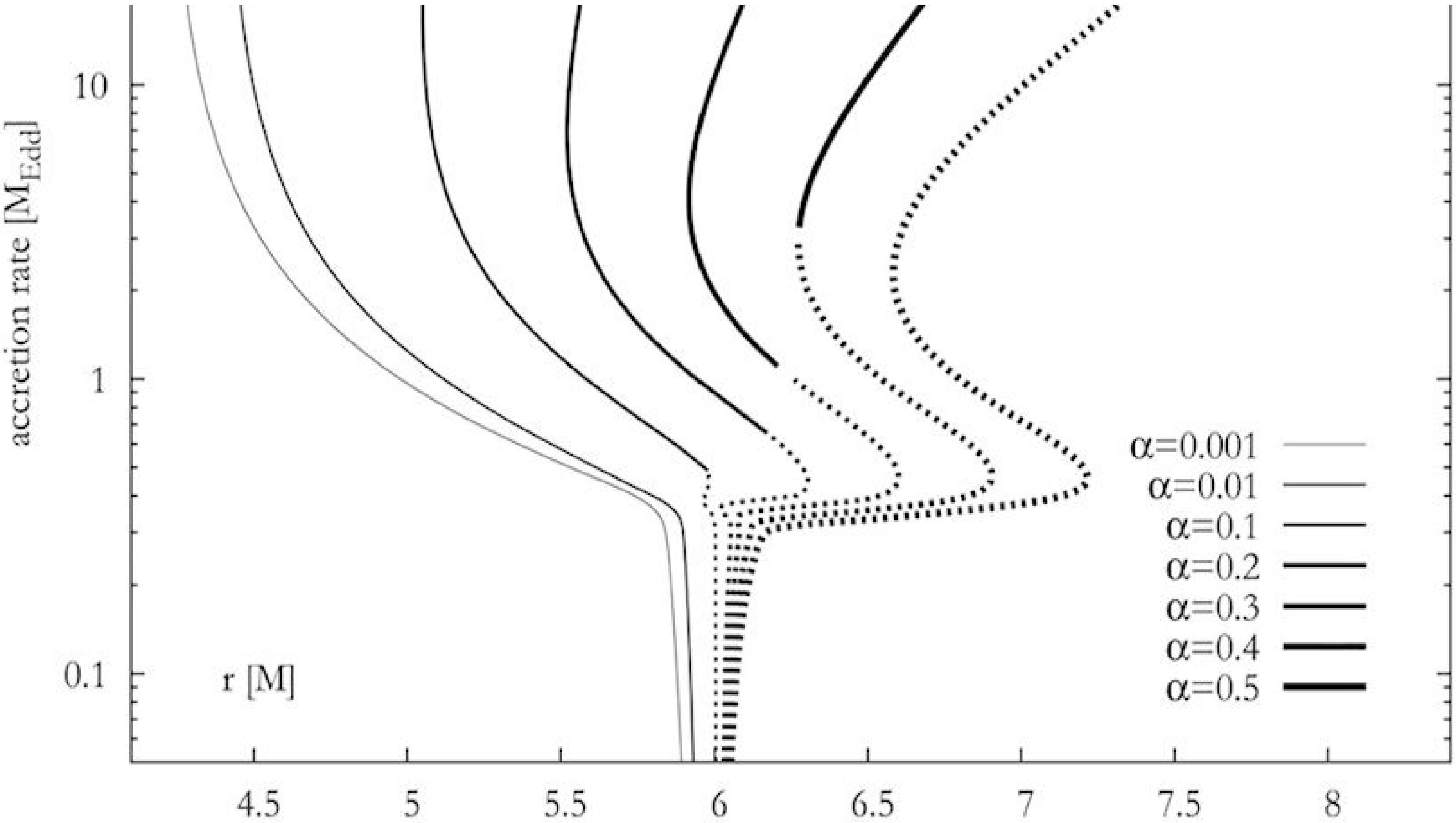


Fig. 7. Location of the sonic point as a function of the accretion rate for different values of α, for a non-rotating black hole, $a^* = 0$. The solid curves are for saddle-type solutions, while the dotted curves present nodal-type regimes.

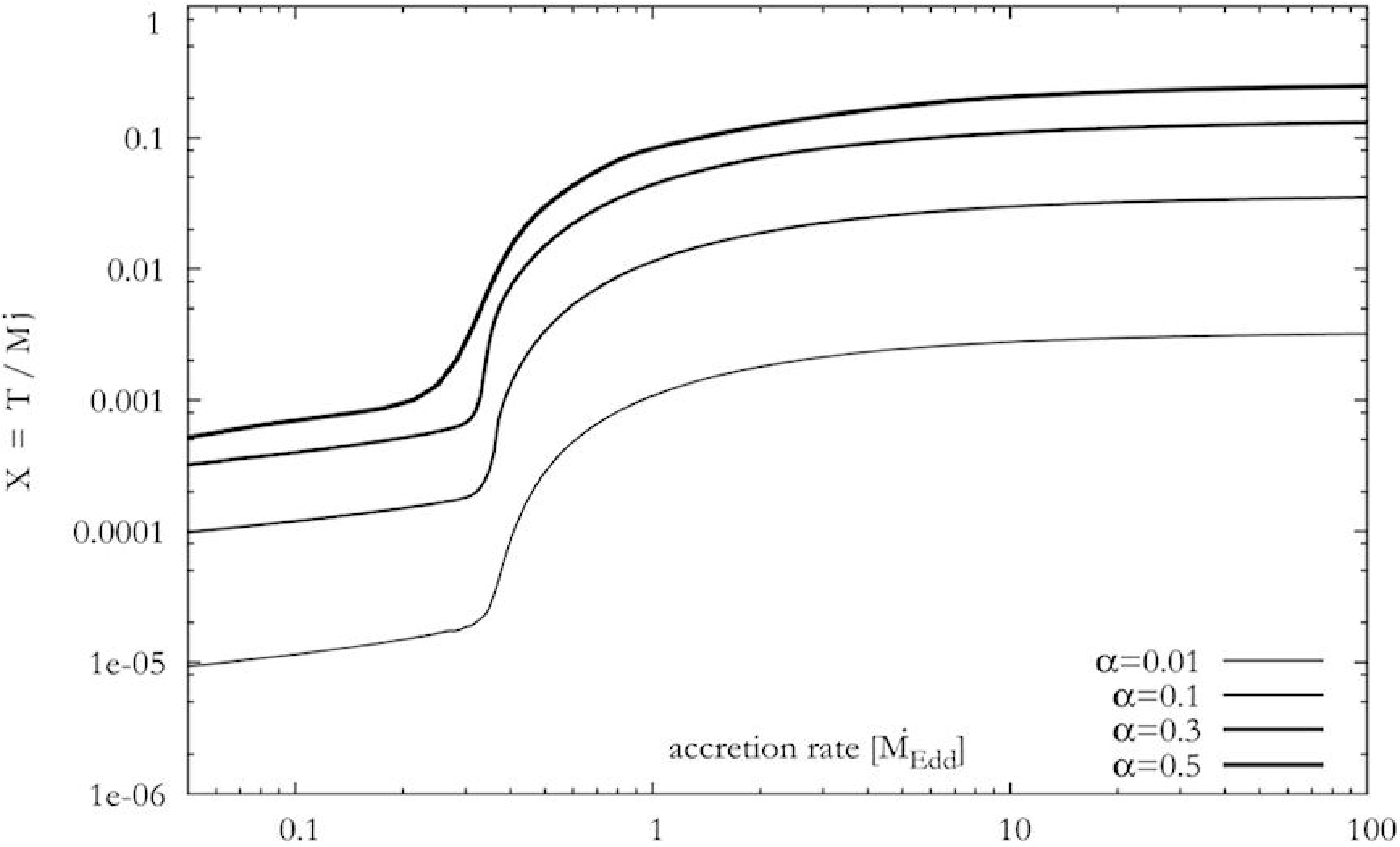


Fig. 11. Ratio of the angular momentum flux caused by torque to the flux caused by advection calculated at r_{son}

An idea on the black hole accretion